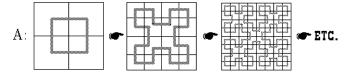


Spacefilling Curve

- a continuous mapping of the unit interval [0,1] onto a unit hypercube (for example, a square)
- first introduced by Peano (1891), Hilbert (1891), and others as "topological monsters"
- tend to preserve "nearness" among points:
 - if 2 points are close on the curve, they are close in the square
 - if 2 points are close in the square, they are "likely" to be close on the curve.

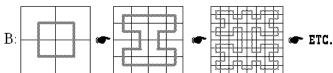
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Examples...



Each curve is the limit of a sequence of recursive constructions.

Examples...

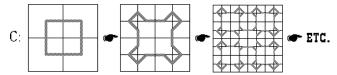


Each curve is the limit of a sequence of recursive constructions.

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Examples...



Each curve is the limit of a sequence of recursive constructions.

"Generic" Spacefilling Curve Heuristic:

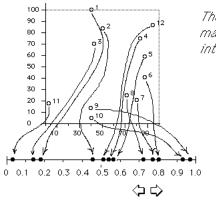
- 1. Transform the problem (routing, location, etc.) in the plane, via a spacefilling curve, to a problem on the unit interval.
- 2. Solve the (easier) problem on the unit interval.
- 3. Map the solution back to the problem in the plane.

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Pseudo-code to compute position on the space-filling curve (A) of a point (x,y) in the unit square:

Random Symmetric TSP Example (seed= 133398) 100 90 o 12 o² 80 70 o⁵ 50 40 30 20 10 o 10 ʹͻເϽ



The square is mapped into the interval [0,1]

Coordinate on Space-filling Curve

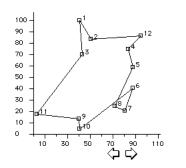
i	Χi	Υi	Position	
1 2 3 4 5 6 7 8 9 10 11 12	41 51 43 84 88 81 72 40 41 3	100 84 70 75 59 41 21 25 14 5	0.18236921 0.45818231 0.13481542 0.52611921 0.55589939 0.77961296 0.71850459 0.96073832 0.93535641 0.02027783 0.51850270	
→				

Sort the points:

i	Position
11	0.02027783
3	0.13481542
1	0.18236921
2	0.45818231
12	0.51850270
4	0.52611921
5	0.55589939
8	0.71850459
7	0.77961296
6	0.80589939
10	0.93538641
9	0.96073832

This determines the order in which cities are visited...

Space-filling-curve tour: 1 2 12 4 5 8 7 6 10 9 11 3 1 with length 365



The space-filling curve has been suggested as a "low-technology" algorithm...

- 1) Find location on map and read (x,y) coordinates
- 2) Find (x,y) in table giving position
- Prepare card with (x,y) and position, and insert into card file according to the position
- 4) The sorted cards give a tour.

[&]quot;A Minimal Technology Routing System for Meals on Wheels", Interfaces, June 1983 (Volume 13, no. 2), by J. Bartholdi et al.

