

Simulated Annealing

Simulated annealing is similar to Lin's k-exchange algorithm, except that an exchange that results in an increase in the tour length is accepted with a positive probability. (This probability varies inversely with the magnitude of the increase, and for a given increase, decreases as the algorithm progresses.)

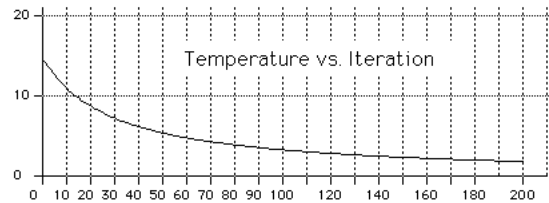
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"Simulated Annealing"

- a heuristic search approach
- a move is made to any neighboring solution with equal or lower cost
- if the neighbor increases the cost by $\Delta > 0$, then the move is accepted with probability $P\{\text{accept } \Delta\} = e^{-\Delta/T}$ where T is the current "temperature" of the system
- the system is "cooled" according to some "cooling schedule"



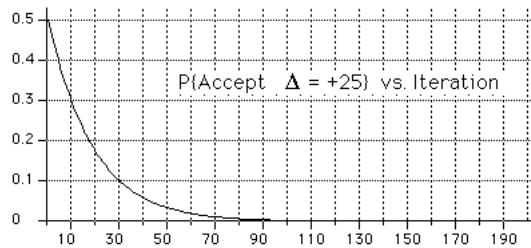
After each iteration, the temperature is reduced, according to a "cooling schedule"



$$T_{i+1} = \frac{T_i}{1 + \beta T_i} \quad \text{where } \beta = \frac{(T_0 - T_f)}{M T_0 T_f} \quad \& \quad \begin{cases} T_0 = \text{initial temperature} \\ T_f = \text{final temperature} \\ M = \# \text{ of iterations} \end{cases}$$



As the system "cools", the probability of accepting an increase (of 25) decreases:



$$P\{\text{accept } \Delta\} = e^{-\Delta/T}$$



Simulated Annealing (starting with random tour)

Temperature will decrease from 36.067 to 2.171
Cooling parameter beta is 0.002163955657

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Tour # 1 is 1 / 11 7 / 4 2 3 8 10 5 12 9 6 1
Length: 685 Improvement: 44
Tour # 2 is 1 / 4 7 11 / 2 3 8 10 5 12 9 6 1
Length: 693 Improvement: 8
Tour # 3 is 1 / 2 11 7 4 / 3 8 10 5 12 9 6 1
Length: 687 Improvement: 6
Tour # 4 is 1 / 3 4 7 11 2 / 8 10 5 12 9 6 1
Length: 707 Improvement: 20
Tour # 5 is 1 / 12 5 10 8 2 11 7 4 3 / 9 6 1
Length: 698 Improvement: 9
Tour # 6 is 1 / 9 3 4 7 11 2 8 10 5 12 / 6 1
Length: 720 Improvement: 22
Tour # 7 is 1 9 / 7 4 3 / 11 2 8 10 5 12 6 1
Length: 694 Improvement: 26
Tour # 8 is 1 9 / 11 3 4 7 / 2 8 10 5 12 6 1
Length: 677 Improvement: 17
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Best Tour: 1 2 3 11 9 10 8 7 6 5 4 12 1, with length 321

CPU time for simulated annealing: 123.35 seconds

