

This can be a truly huge LP to solve!

11/3/00

Stochastic LP via Benders
Benders decomposition partitions the variables into
scenarios having discrete values. For each, the number
of scenarios is $b + 1 \leq b$, while the number of rows
in the case $x_1 = x_2 = \dots = x_n = 0$ is b .

Minimize $c_1x_1 + p_1y_1 + p_2y_2$ subject to
subject to $Ax_1 + B_1y_1 + B_2y_2 = b$
is equivalent to the master problem
and $Ax_1 + B_1y_1 + B_2y_2 = b$
the limitations
That is, for any x , we evaluate $w(x)$ by adding the
1st stage cost c_1x to the sum of the stage LP
solutions given a trial 1st stage
solution by the master problem.

Stochastic LP with Recourse



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Objective

Consider again the stochastic problem of the golf-bag manufacturer (Par, Inc.)

$$\begin{aligned} \text{Max } & 10X_1 + 9X_2 + 0.3(8Y_1^0 - 5T_{CD}^0 - 6T_S^0 - 8T_F^0 - 4T_{IP}^0) \\ & + 0.3(8Y_1^1 - 5T_{CD}^1 - 6T_S^1 - 8T_F^1 - 4T_{IP}^1) \\ & + 0.3(8Y_1^2 - 5T_{CD}^2 - 6T_S^2 - 8T_F^2 - 4T_{IP}^2) \\ & + 0.1(8Y_1^3 - 5T_{CD}^3 - 6T_S^3 - 8T_F^3 - 4T_{IP}^3) \end{aligned}$$

Equivalent Deterministic Linear Programming Model

Stochastic LP via Benders
 The first-stage problem is a linear program with the objective function $z_1(X)$ and constraints $A_1 X \leq b_1$. The second-stage problem is a linear program with the objective function $z_2(X, U)$ and constraints $A_2(X) U \leq b_2(X)$. The overall problem is a linear program with the objective function $z(X) = z_1(X) + \sum_{i=1}^4 p_i z_2(X, U_i)$ and constraints $A_1 X \leq b_1$. The trial values of X are $X_1 = 360.907740$ and $X_2 = 295.636900$. The overall problem is linear in X , and can be solved with CPLEX. The trial values of X are used to solve the second-stage problem for each scenario. The overall problem is linear in X , and can be solved with CPLEX. The trial values of X are used to solve the second-stage problem for each scenario.

Linear in X , for fixed values of U

Now, using the trial values of X , namely

- X1 360.907740
- X2 295.636900

we solve the second-stage problem for each of the four scenarios:

- Scenario #0 Company fails to obtain both contracts
- Scenario #1 Company wins contract #1, loses #2
- Scenario #2 Company wins contract #2, loses #1
- Scenario #3 Company wins both contracts #1 & #2

Click to obtain solution for each scenario! ➔

Stochastic LP via Benders
SCENARIO 0 CONTRACTS
 ROW 10 SLACK OR DUAL
 MAX 8 Y10 - 5 TCD0 - 6 TS0
 SUBJECT TO
 2) 0.7 Y10 - TCD0 <= 31.723
 3) 0.5 Y10 - TS0 <= 133.178
 4) Y10 - TF0 <= 69.997
 5) 0.1 Y10 - TIP0 <= 15
 6) TF0 <= 100
 END
 production schedule of 360.9 standard and 295.6 deluxe golf bags.



RECOURSE

That is, if the initial trial solution $X = (360.9, 295.6)$ were chosen, and scenario 0 occurs, i.e., neither bid was successful, then some capacity remains idle in each of the departments... the optimal recourse would be to schedule production of an additional 98.2 standard golf bags and an additional 23.275 hours in the cutting & dyeing department.



SCENARIO 0

SCENARIO 1

company wins 1st contract, loses 2nd contract

MAX 8 Y11 - 5 TCD1 - 6 TS1
 - 8 TF1 - 4 TIP1
 SUBJECT TO
 2) 0.7 Y11 - TCD1 <= 31.723
 3) 0.5 Y11 - TS1 <= 133.178
 4) Y11 - TF1 <= 69.997
 5) 0.1 Y11 - TIP1 <= 15
 6) TF1 <= 100
 END



RECOURSE

That is, if the trial stage-1 solution $X = (360.9, 295.6)$ were used, and the first bid (but not the second) were successful, each of the four departments will have excess capacity... the optimal recourse is to schedule production of an additional 69.997 standard golf bags, and use of 17.275 hours of overtime in the cutting & dyeing department.

SCENARIO 1

Scenario 1: Via Bender's loses 1st contract, wins 2nd contract

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OBJECTIVE FUNCTION VALUE

618.6015

That is, the company should

VARIABLE	VALUE	REDUCED COST
TCD3	0.0000	0.0000
TS3	0.0000	0.0000
TF3	0.0000	0.0000
TIP3	0.0000	0.0000
TIP2	4.0000	4.5000
5)	8.0002	.0000
6)	100.0000	.0000

RECORSE

SCENARIO 2 company loses 1st contract, wins 2nd contract

MAX 8 Y12 - 5 TCD2 - 6 TS2 - 8 TF2 - 4 TIP2

SUBJECT TO

2) 0.7 Y12 - TCD2 <= 51.723

3) 0.5 Y12 - TS2 <= 123.178

4) Y12 - TF2 <= 79.997

5) 0.1 Y12 - TIP2 <= 10

6) TF2 <= 100

END



SCENARIO 1



ROW	SLACK OR SURPLUS	DUAL PRICES
2)	.0000	5.0000
3)	83.1795	.0000
4)	.0000	4.5000
5)	2.0003	.0000
6)	100.0000	.0000



SCENARIO 2



SCENARIO 3 company wins both contracts!

MAX 8 Y13 - 5 TCD3 - 6 TS3 - 8 TF3 - 4 TIP3

SUBJECT TO

2) 0.7 Y3 - TCD3 <= 1.723

3) 0.5 Y3 - TS3 <= 83.178

4) Y3 - TF3 <= 0

5) 0.1 Y3 - TIP3 <= 0

6) TF3 <= 100

END

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	1.7230	.0000
3)	83.1780	.0000
4)	.0000	8.0000
5)	.0000	.0000
6)	100.0000	.0000

SCENARIO 3

11/3/00

Master Problem $z = 10X_1 + 12X_2$ $z = 10(360.9) + 12(295.6) = 6269.81$

SCENARIO 3

U₁ = 0.00015, U₂ = 0.00015, U₃ = 0.00015, U₄ = 0.00015

U₅ = 0.00015, U₆ = 0.00015, U₇ = 0.00015, U₈ = 0.00015

U₉ = 0.00015, U₁₀ = 0.00015, U₁₁ = 0.00015, U₁₂ = 0.00015

U₁₃ = 0.00015, U₁₄ = 0.00015, U₁₅ = 0.00015, U₁₆ = 0.00015

U₁₇ = 0.00015, U₁₈ = 0.00015, U₁₉ = 0.00015, U₂₀ = 0.00015

U₂₁ = 0.00015, U₂₂ = 0.00015, U₂₃ = 0.00015, U₂₄ = 0.00015

U₂₅ = 0.00015, U₂₆ = 0.00015, U₂₇ = 0.00015, U₂₈ = 0.00015

U₂₉ = 0.00015, U₃₀ = 0.00015, U₃₁ = 0.00015, U₃₂ = 0.00015

U₃₃ = 0.00015, U₃₄ = 0.00015, U₃₅ = 0.00015, U₃₆ = 0.00015

U₃₇ = 0.00015, U₃₈ = 0.00015, U₃₉ = 0.00015, U₄₀ = 0.00015

U₄₁ = 0.00015, U₄₂ = 0.00015, U₄₃ = 0.00015, U₄₄ = 0.00015

U₄₅ = 0.00015, U₄₆ = 0.00015, U₄₇ = 0.00015, U₄₈ = 0.00015

U₄₉ = 0.00015, U₅₀ = 0.00015, U₅₁ = 0.00015, U₅₂ = 0.00015

U₅₃ = 0.00015, U₅₄ = 0.00015, U₅₅ = 0.00015, U₅₆ = 0.00015

U₅₇ = 0.00015, U₅₈ = 0.00015, U₅₉ = 0.00015, U₆₀ = 0.00015

U₆₁ = 0.00015, U₆₂ = 0.00015, U₆₃ = 0.00015, U₆₄ = 0.00015

U₆₅ = 0.00015, U₆₆ = 0.00015, U₆₇ = 0.00015, U₆₈ = 0.00015

U₆₉ = 0.00015, U₇₀ = 0.00015, U₇₁ = 0.00015, U₇₂ = 0.00015

U₇₃ = 0.00015, U₇₄ = 0.00015, U₇₅ = 0.00015, U₇₆ = 0.00015

U₇₇ = 0.00015, U₇₈ = 0.00015, U₇₉ = 0.00015, U₈₀ = 0.00015

U₈₁ = 0.00015, U₈₂ = 0.00015, U₈₃ = 0.00015, U₈₄ = 0.00015

U₈₅ = 0.00015, U₈₆ = 0.00015, U₈₇ = 0.00015, U₈₈ = 0.00015

U₈₉ = 0.00015, U₉₀ = 0.00015, U₉₁ = 0.00015, U₉₂ = 0.00015

U₉₃ = 0.00015, U₉₄ = 0.00015, U₉₅ = 0.00015, U₉₆ = 0.00015

U₉₇ = 0.00015, U₉₈ = 0.00015, U₉₉ = 0.00015, U₁₀₀ = 0.00015

The expected profit if $X=(360.9, 295.6)$ were selected as the values of the stage-1 variables, is

profit	
6269.81	1st stage
0.3x1083.6	scenario 0
0.3x 473.6	scenario 1
0.3x 618.6	scenario 2
0.1x 0	scenario 3
6922.6	TOTAL

We now compute this objective for hours value of depts.

RECOURSE



OBJECTIVE FUNCTION VALUE

1416.000

VARIABLE	VALUE	REDUCED COST
X1	708.00	.0000
X2	.00	.0666

Right-Hand-Sides of 2nd Stage Problems

VARIABLE	VALUE	REDUCED COST
Y0	0	5.000000
TCDO	134.400000	104.400000
TSO	0	196.000000
TFO	64.200000	94.200000
TIPO	45	109.000000
ART	100	100

Now, using the trial values of X, namely

X1 708.00
X2 .00

we solve the second-stage problem for each of the four scenarios:

- Scenario #0 Company fails to obtain both contracts
- Scenario #1 Company wins contract #1, loses #2
- Scenario #2 Company wins contract #2, loses #1
- Scenario #3 Company wins both contracts #1 & #2

Click to obtain solution for each scenario! →

SCENARIO 0 company loses both contracts

```

MAX 8 Y0 - 5 TCDO - 6 TSO
      - 8 TFO - 4 TIPO - 100 ART
SUBJECT TO
2) 0.7 Y0 - TCDO - ART <= 134.4
3) 0.5 Y0 - TSO - ART <= 0
4) Y0 - TFO - ART <= 64.2
5) 0.1 Y0 - TIPO - ART <= 45
6) TFO <= 100
END
    
```



ROW	SLACK OR SURPLUS	DUAL PRICES
2)	89.460000	.000000
3)	.000000	6.000000
4)	.000000	5.000000
5)	38.580001	.000000
6)	100.000000	.000000

SCENARIO 1 company wins 1st contract, loses 2nd contract

```

MAX 8 Y1 - 5 TCD1 - 6 TS1
      - 8 TF1 - 4 TIP1 - 100 ART
SUBJECT TO
2) 0.7 Y1 - TCD1 - ART <= 84.4
3) 0.5 Y1 - TS1 - ART <= 206
4) Y1 - TF1 - ART <= - 80
5) 0.1 Y1 - TIP1 - ART <= 54.2
6) TF1 <= 100
END
    
```



ROW	SLACK OR SURPLUS	DUAL PRICES
2)	84.400001	.000000
3)	206.000000	.000000
4)	.000000	8.000000
5)	54.200000	.000000
6)	20.000000	.000000

1) =580.0000

SCENARIO 2
 MAX 0.000000 - 5 TCD3 - 6 TS3
 - 70.000000 - 8 TF2 - 100 ART
 SUBJECT TO
 2) 0.7 Y2 - TCD3 - ART <= 104.4
 3) 0.5 Y2 - TS2 - ART <= 196
 4) Y2 - TF2 - ART <= -70
 5) 0.1 Y2 - TIP2 - ART <= 49.2
 6) TF2 <= 100
 END



ROW	SLACK OR SURPLUS	DUAL PRICES
2)	104.400000	.000000
3)	196.000000	.000000
4)	.000000	8.000000
5)	49.200000	.000000
6)	30.000000	.000000

SCENARIO 3 company wins both contracts!

MAX 8 Y3 - 5 TCD3 - 6 TS3
 - 8 TF3 - 4 TIP3 - 100 ART
 SUBJECT TO
 2) 0.7 Y3 - TCD3 - ART <= 54.4
 3) 0.5 Y3 - TS3 - ART <= 156
 4) Y3 - TF3 - ART <= -150
 5) 0.1 Y3 - TIP3 - ART <= 39.2
 6) TF3 <= 100
 END



ROW	SLACK OR SURPLUS	DUAL PRICES
2)	104.400000	.000000
3)	206.000000	.000000
4)	.000000	100.000000
5)	89.200000	.000000
6)	.000000	92.000000

The expected profit if $X=(708, 0)$ were selected as the values of the stage-1 variables, is

profit		
	7080	1st stage
0.3x	321.00	scenario 0
0.3x	-640.00	scenario 1
0.3x	-560.00	scenario 2
0.1x	-5800.00	scenario 3
	6236.3	TOTAL



DUAL PRICES

The optimal dual variables from the subproblem will be used to compute a linear approximation to $v(X)$

	scenario			
	0	1	2	3
U_1	.000	.000	.000	.000
U_2	6.000	.000	.000	.000
U_3	5.000	8.000	8.000	100.000
U_4	.000	.000	.000	.000
U_5	.000	.000	.000	92.000

the subproblem. We obtain a linear function in the first stage variables X_1 and X_2 :

$$\begin{aligned} \text{Max } & 2.7X_1 + 1.2X_2 + 115.89 \\ \text{subject to } & 0.31U_1 + 0.3333U_2 + 0.6667U_3 + 0.25U_4 \\ & -0.3U_1 + 0.3333U_2 + 0.6667U_3 + 0.25U_4 \end{aligned}$$

Note: Evaluating the linear function at $X^1 = (709, 0)$, the fixed values of the first-stage problem, gives 6582.4 which agrees with the total expected profit from stages 1-40, rounded to 1000, that is, this approximation is close to 1000.

no!
We now compute this objective for fixed values of U!

OBJECTIVE FUNCTION VALUE

1) 7172.817

VARIABLE	VALUE	REDUCED COST
X1	492.6717	.000000
X2	285.1298	.000000
Z	7172.8173	.000000

That is, our next trial solution is 492.67 standard and 285.13 deluxe bags. \$7172.81 is an upper bound on the maximum profit!