Benders' Decomposition Applied to Stochastic Linear Programming with Recourse

Stochastic LP with Recourse

author

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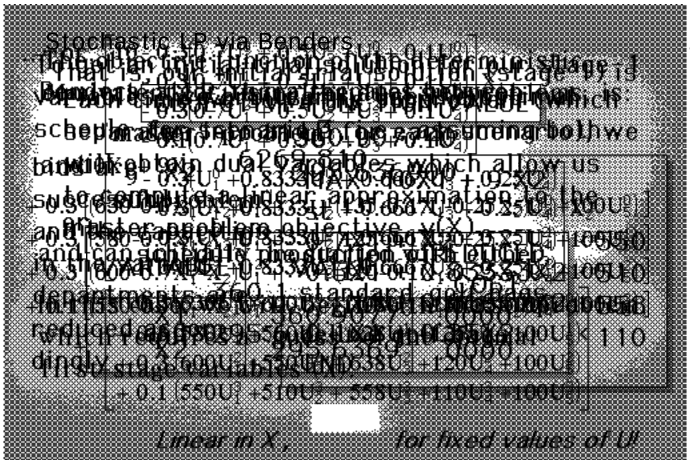
Objective

Consider again the stochastic problem of the golf-bag manufacturer (Par, Inc.)

$$
\begin{aligned}
\text{Max } & 10X_1 + 9X_2 + 0.3\left(8Y_1^0 - 5T_{CD}^0 - 6T_S^0 - 8T_F^0 - 4T_{IP}^0\right) \\
& + 0.3\left(8Y_1^1 - 5T_{CD}^1 - 6T_S^1 - 8T_F^1 - 4T_{IP}^1\right) \\
& + 0.3\left(8Y_1^2 - 5T_{CD}^2 - 6T_S^2 - 8T_F^2 - 4T_{IP}^2\right) \\
& + 0.1\left(8Y_1^3 - 5T_{CD}^3 - 6T_S^3 - 8T_F^3 - 4T_{IP}^3\right)
\end{aligned}
$$

Equivalent Deterministic Linear Programming Model



Now, using the trial values of X, namely

X1	360.907740
X2	295.636900

we solve the second-stage problem for each of the four scenarios:

Scenario #0	Company fails to obtain both contracts
Scenario #1	Company wins contract #1, loses #2
Scenario #2	Company wins contract #2, loses #1
Scenario #3	Company wins both contracts #1 & #2

Click to obtain solution for each scenario!

SCENARIO 0 company loses both contracts

OBJECTIVE FUNCTION VALUE

MAX 8 Y10 - 5 TCD0 - 6 TS0

SUBJECT TO

END

In this case, the right-hand sides (the capacities in the various departments) are the original departmental capacities, minus the amounts of the capacities used by the trial stage-1 production schedule of 360.9 standard and 295.6 deluxe golf bags.

SCENARIO 0

RECOURSE

That is, if the initial trial solution
X = (360.9, 295.6)
were chosen, and scenario 0 occurs, i.e., neither bid was successful, then some capacity remains idle in each of the departments... the optimal recourse would be to schedule production of an additional 98.2 standard golf bags and an additional 23.275 hours in the cutting & dyeing department.

SCENARIO 0

SCENARIO 1 company wins 1st contract, loses 2nd contract

```
MAX     8 Y11 - 5 TCD1 - 6 TS1
                - 8 TF1 - 4 TIP1
SUBJECT TO
     2)  0.7 Y11 - TCD1 <=   31.723
     3)  0.5 Y11 - TS1  <=  133.178
     4)  Y11 - TF1      <=   69.997
     5)  0.1 Y11 - TIP1 <=   15
     6)           TF1   <=  100
END
```

RECOURSE

That is, if the trial stage-1 solution
X = (360.9, 295.6)
were used, and the first bid (but not the second) were successful, each of the four departments will have excess capacity...
the optimal recourse is to schedule production of an additional 69.997 standard golf bags, and use of 17.275 hours of overtime in the cutting & dyeing department.

SCENARIO 1

SCENARIO 2 company loses 1st contract, wins 2nd contract

OBJECTIVE FUNCTION VALUE

618.6015

[illegible]

RECOURSE

SCENARIO 1

SCENARIO 2 company loses 1st contract, wins 2nd contract

```
MAX    8 Y12 - 5 TCD2 - 6 TS2
              - 8 TF2   - 4 TIP2
SUBJECT TO
     2)  0.7 Y12 - TCD2 <=   51.723
     3)  0.5 Y12 - TS2 <=  123.178
     4)  Y12 - TF2 <=   79.997
     5)  0.1 Y12 - TIP2 <=   10
     6)  TF2 <=   100
END
```

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	.0000	5.0000
3)	83.1795	.0000
4)	.0000	4.5000
5)	2.0003	.0000
6)	100.0000	.0000

SCENARIO 2

SCENARIO 3 company wins both contracts!

```
MAX    8 Y13 - 5 TCD3 - 6 TS3
              - 8 TF3   - 4 TIP3
SUBJECT TO
     2)  0.7 Y3   - TCD3 <=    1.723
     3)  0.5 Y3   - TS3  <=  83.178
     4)  Y3  - TF3       <=    0
     5)  0.1 Y3   - TIP3 <=    0
     6)              TF3 <= 100
END
```

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	1.7230	.0000
3)	83.1780	.0000
4)	.0000	8.0000
5)	.0000	.0000
6)	100.0000	.0000

SCENARIO 3

[illegible]

We now compute this objective for fixed values of U.

RECOURSE

The expected profit *if* X=(360.9, 295.6) were selected as the values of the stage-1 variables, is

profit	
6269.81	1st stage
0.3x1083.6	scenario 0
0.3x 473.6	scenario 1
0.3x 618.6	scenario 2
0.1x 0	scenario 3
6922.6	TOTAL

OBJECTIVE FUNCTION VALUE

1416.000

VARIABLE	VALUE	REDUCED COST
X1	708.00	.0000
X2	.00	.0666

Right-Hand-Sides of 2nd Stage Problems

row	scenario 0	1	2	3
1	134.4	84.4	[illegible]	34.4
2	[illegible]	[illegible]	196	[illegible]
3	0	-80	[illegible]	-150
4	64.2	[illegible]	[illegible]	[illegible]
5	100	100	100	100

Now, using the trial values of X, namely

X1	708.00
X2	.00

we solve the second-stage problem for each of the four scenarios:

Scenario #0 Company fails to obtain both contracts

Scenario #1 Company wins contract #1, loses #2

Scenario #2 Company wins contract #2, loses #1

Scenario #3 Company wins both contracts #1 & #2

Click to obtain solution for each scenario!

SCENARIO 0 company loses both contracts

```
MAX     8 Y0   - 5 TCD0 - 6 TS0
               - 8 TF0 - 4 TIP0 - 100 ART
SUBJECT TO
     2)  0.7 Y0   - TCD0 - ART <=   134.4
     3)  0.5 Y0   - TS0  - ART <=     0
     4)  Y0   - TF0      - ART <=    64.2
     5)  0.1 Y0   - TIP0 - ART <=    45
     6)  TF0                   <=   100
END
```

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	89.460000	.000000
3)	.000000	6.000000
4)	.000000	5.000000
5)	38.580001	.000000
6)	100.000000	.000000

SCENARIO 1 company wins 1st contract, loses 2nd contract

```
MAX     8 Y1   - 5 TCD1 - 6 TS1
               - 8 TF1 - 4 TIP1 - 100 ART
SUBJECT TO
     2)  0.7 Y1   - TCD1 - ART <=    84.4
     3)  0.5 Y1   - TS1  - ART <=   206
     4)  Y1   - TF1      - ART <=   - 80
     5)  0.1 Y1   - TIP1 - ART <=    54.2
     6)  TF1                   <=   100
END
```

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	84.400001	.000000
3)	206.000000	.000000
4)	.000000	8.000000
5)	54.200000	.000000
6)	20.000000	.000000

OBJECTIVE FUNCTION VALUE

1) -5800.000

VARIABLE	VALUE	REDUCED COST
Y3	.000000	92.000000
TCD3	.000000	5.000000
TS3	.000000	6.000000
TF3	100.000000	.000000
TIP3	.000000	4.000000
ART	.000000	92.000000

SCENARIO 2 company wins 1st contract, loses 2nd contract

```
MAX   8 Y2  - 5 TCD2 - 6 TS2
            - 8 TF2 - 4 TIP2 - 100 ART
SUBJECT TO
   2)  0.7 Y2  - TCD2  - ART <=  104.4
   3)  0.5 Y2  - TS2   - ART <=  196
   4)  Y2  - TF2       - ART <=  - 70
   5)  0.1 Y2  - TIP2  - ART <=   49.2
   6)  TF2                   <=  100
END
```

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	104.400000	.000000
3)	196.000000	.000000
4)	.000000	8.000000
5)	49.200000	.000000
6)	30.000000	.000000

SCENARIO 3 company wins both contracts!

```
MAX   8 Y3  - 5 TCD3 - 6 TS3
            - 8 TF3 - 4 TIP3 - 100 ART
SUBJECT TO
   2)  0.7 Y3  - TCD3 - ART <=   54.4
   3)  0.5 Y3  - TS3  - ART <=  156
   4)  Y3  - TF3      - ART <= - 150
   5)  0.1 Y3  - TIP3 - ART <=   39.2
   6)  TF3                  <=  100
END
```

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	104.400000	.000000
3)	206.000000	.000000
4)	.000000	100.000000
5)	89.200000	.000000
6)	.000000	92.000000

The expected profit *if* X=(708, 0) were selected as the values of the stage-1 variables, is

profit	
7080	1st stage
0.3x 321.00	scenario 0
0.3x ⁻640.00	scenario 1
0.3x ⁻560.00	scenario 2
0.1x⁻5800.00	scenario 3
6236.3	TOTAL

DUAL PRICES The optimal dual variables from the subproblem will be used to compute a linear approximation to v(X)

	scenario 0	scenario 1	scenario 2	scenario 3
U_1	.000	.000	.000	.000
U_2	6.000	.000	.000	.000
U_3	5.000	8.000	8.000	100.000
U_4	.000	.000	.000	.000
U_5^0	.000	.000	.000	92.000

Master Problem

... Benders ... variables which optimize the subproblem(s), we obtain a linear function in the first-stage variables, X_1 and X_2:

Max z

subject to

$z \leq 2X_1 + 3.3667 X_2 + 5826.3$

☞ *Note: Evaluating the linear function at $X^1 = (708, 0)$, the fixed values of the first-stage problem, gives 6582.4 which agrees with the total expected profit from stages 1 & 2 found earlier! That is, this "approximation" is exact at X^1!*

no! We now compute this objective for fixed values of U!

OBJECTIVE FUNCTION VALUE

1) 7172.817

VARIABLE	VALUE	REDUCED COST
X1	492.6717	.000000
X2	285.1298	.000000
Z	7172.8173	.000000

That is, our next trial solution is 492.67 standard and 285.13 deluxe bags. $7172.81 is an *upper* bound on the maximum profit!