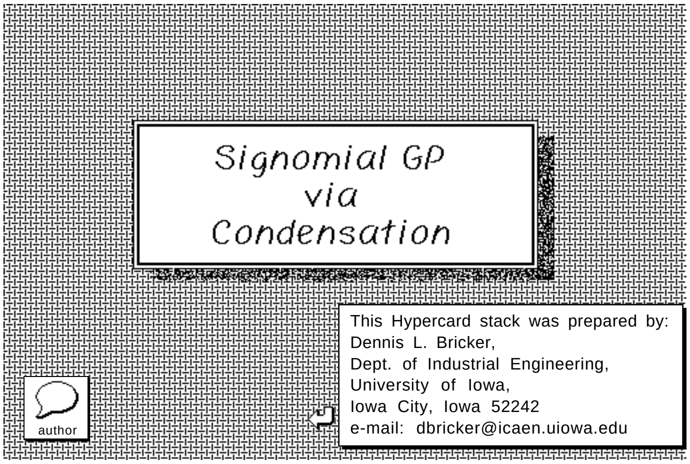
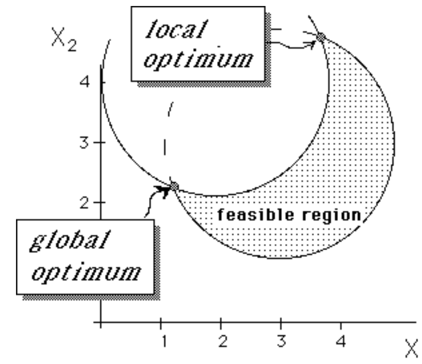




Reformulation as a GP problem

constraint # 1

$$(X_1 - 2)^2 + (X_2 - 4)^2 \ge 4$$

$$\Rightarrow \quad (x_1^2 - 4x_1 + 4) + (x_2^2 - 8x_2 + 16) \ge 4$$

$$\Rightarrow \quad -x_1^2 + 4x_1 - x_2^2 + 8x_1 \le 16$$

The constraint becomes the signomial constraint

$$\Rightarrow \quad \boxed{\frac{X_1}{4} + \frac{X_2}{2} - \frac{X_1^2}{16} - \frac{X_2^2}{16} \le 1}$$

Reformulation as a GP problem

constraint # 2

$$(X_1 - 3)^2 + (X_2 - 3)^2 \le 4$$

$$\Rightarrow \quad (x_1^2 - 6x_1 + 9) + (x_2^2 - 6x_2 + 9) \le 4$$

$$\Rightarrow \quad x_1^2 - 6x_1 + x_2^2 + 14 \le 6x_2$$

The constraint becomes the signomial constraint

$$\Rightarrow \quad \boxed{\frac{X_1^2X_2^{-1}}{6} + \frac{X_2}{6} + \frac{7X_2^{-1}}{3} - X_1X_2^{-1} \le 1}$$

Signomial Geometric Program

Minimize X_1
subject to

$$\frac{X_1}{4} + \frac{X_2}{2} - \frac{X_1^2}{16} - \frac{X_2^2}{16} \le 1$$

$$\frac{X_1^2X_2^{-1}}{6} + \frac{X_2}{6} + \frac{7X_2^{-1}}{3} - X_1X_2^{-1} \le 1$$

$$X_1 > 0,\ X_2 > 0$$

To condense the signomial constraint

$$\frac{X_1}{4} + \frac{X_2}{2} - \frac{X_1^2}{16} - \frac{X_2^2}{16} \le 1$$

we first write it in the form

$$\frac{X_1}{4} + \frac{X_2}{2} \le 1 + \frac{X_1^2}{16} + \frac{X_2^2}{16}$$

$$\Rightarrow \quad \frac{\frac{X_1}{4} + \frac{X_2}{2}}{1 + \frac{X_1^2}{16} + \frac{X_2^2}{16}} \le 1 \quad \Rightarrow \quad \frac{0.25X_1 + 0.5X_2}{1 + 0.0625\,X_1^2 + 0.0625\,X_2^2} \le 1$$

We next condense the denominator of

$$\frac{0.25X_1 + 0.5X_2}{1 + 0.0625\,X_1^2 + 0.0625\,X_2^2} \le 1$$

into a single term. Let's use the point $X_0 = (4,5)$ at which the terms of the denominator are

1 + 1 + 1.5626 = 3.5625

Then

$$\delta_1 = \delta_2 = \frac{1}{3.5625} = 0.2807 \quad \text{and} \quad \delta_3 = \frac{1.5625}{3.5625} = 0.4386$$

$\delta_1 = \delta_2 = 0.2807, \quad \delta_3 = 0.4386$

Coefficient:

$$\boxed{C(\delta) = \prod_{i=1}^{3} \left(\frac{c_i}{\delta_i}\right)^{\delta_i}}$$

$$C(\delta) = \left(\frac{1}{0.2807}\right)^{0.2807} \left(\frac{0.0625}{0.2807}\right)^{0.2807} \left(\frac{0.0625}{0.4386}\right)^{0.4386}$$

$= 0.3987$

$\delta_1 = \delta_2 = 0.2807, \quad \delta_3 = 0.4386$

Exponents: $a_j(\delta) = \sum_{i=1}^{3} a_{ij}\delta_i$

$$a_1 = 0\delta_1 + 2\delta_2 + 0\delta_3 = 2(0.2807) = 0.5614$$
$$a_2 = 0\delta_1 + 0\delta_2 + 2\delta_3 = 2(0.4386) = 0.8772$$

$C(\delta) = 0.3987$

$a_1 = 0.5614$

$a_2 = 0.8772$

Condensed denominator is

$0.3987\, X_1^{0.5614} X_2^{0.8772}$

monomial!

Geometric Inequality implies

$$1 + 0.0625X_1^2 + 0.0625\, X_2^2 \geq 0.3987\, X_1^{0.5614} X_2^{0.8772}$$

and so

$$\frac{0.25X_1 + 0.5X_2}{1 + 0.0625\, X_1^2 + 0.0625\, X_2^2} \leq \frac{0.25X_1 + 0.5X_2}{0.3987\, X_1^{0.5614} X_2^{0.8772}}$$

$\dfrac{\textit{posynomial}}{\textit{monomial}} = \textit{posynomial}$

$$\frac{0.25X_1 + 0.5X_2}{0.3987\, X_1^{0.5614} X_2^{0.8772}}$$
$$= \frac{0.25}{0.3987} X_1^{1-0.5614} X_2^{-0.8772} + \frac{0.5}{0.3987} X_1^{-0.5614} X_2^{1-0.8772}$$
$$= 0.627\, X_1^{0.4386} X_2^{-0.8772} + 1.254\, X_1^{-0.5614} X_2^{0.1228}$$

which is a posynomial!

If we constrain this posynomial so as to be ≤ 1, then by the geometric inequality, the original signomial should also be ≤ 1.

That is, any X feasible in the posynomial constraint derived by condensation will also be feasible in the signomial constraint:

$$\frac{0.25X_1 + 0.5X_2}{1 + 0.0625\, X_1^2 + 0.0625\, X_2^2} \leq 0.627\, X_1^{0.4386} X_2^{-0.8772} + 1.254\, X_1^{-0.5614} X_2^{0.1228} \leq 1$$

The second signomial constraint may be condensed in a similar fashion:

$$\frac{X_1^2X_2^{-1}}{6} + \frac{X_2}{6} + \frac{7X_2^{-1}}{3} - X_1X_2^{-1} \leq 1$$
$$\Longrightarrow \frac{X_1^2X_2^{-1}}{6} + \frac{X_2}{6} + \frac{7X_2^{-1}}{3} \leq 1 + X_1X_2^{-1}$$
$$\Longrightarrow \frac{\dfrac{X_1^2X_2^{-1}}{6} + \dfrac{X_2}{6} + \dfrac{7X_2^{-1}}{3}}{1 + X_1X_2^{-1}} \leq 1$$

$$\frac{\dfrac{X_1^2X_2^{-1}}{6} + \dfrac{X_2}{6} + \dfrac{7X_2^{-1}}{3}}{1 + X_1X_2^{-1}} \leq 1$$

At (4,5), the denominator is 1 + 0.8 = 1.8, so

$$\delta_1 = \frac{1}{1.8} = 0.555, \; \delta_2 = \frac{0.8}{1.8} = 0.444$$

can be condensed (using $\delta_1 = 0.555$, $\delta_2 = 0.444$) into the posynomial constraint

$$0.08385X_1^{1.555}X_2^{-0.555} + 0.08385X_1^{-0.444}X_2^{1.444} + 1.174X_1^{-0.444}X_2^{-0.555} \leq 1$$

The signomial GP problem is therefore approximated by the posynomial problem:

Minimize X_1

subject to

$$0.627\, X_1^{0.4386} X_2^{-0.8772} + 1.254\, X_1^{-0.5614} X_2^{0.1228} \leq 1$$

$$0.08385X_1^{1.555}X_2^{-0.555} + 0.08385X_1^{-0.444}X_2^{1.444} + 1.174X_1^{-0.444}X_2^{-0.555} \leq 1$$

$$X_1 > 0, \; X_2 > 0$$

Number of variables: 2
Number of polynomials: 3
Total number of terms: 9
Degrees of difficulty: 6
Terms per polynomial: 1 4 4

Coefficients and exponent matrix:

t	p	Ct	expon	
1	1	1	1	0
2	2	0.25	1	0
3	2	0.5	0	1
4	2	¯0.0625	2	0
5	2	¯0.0625	0	2
6	3	0.166667	2	¯1
7	3	0.166667	0	1
8	3	2.33333	0	¯1
9	3	¯1	1	¯1

t = term number, p = polynomial
Ct = coefficient

Sigmonial Problem at x = 4 5

Values of terms (Ut)

t	p	Ut
1	1	4
2	2	1
3	2	2.5
4	2	¯1
5	2	¯1.5625
6	3	0.533333
7	3	0.833333
8	3	0.466667
9	3	¯0.8

Objective function = 4
Constraint function values: 0.9375 1.03333
Infeasibilities: 0 0.0333333

Weights of negative terms, used for condensation:
ρ= 0.280702 0.438596 0.444444

Condensation of Sigmonial GP at x= 4 5

Number of variables: 2
Number of posynomials: 3
Total number of terms: 6
Degrees of difficulty: 3
Terms per posynomial: 1 2 3

Coefficients and exponent matrix:

t	p	Ct	exponents	
1	1	1	1	0
2	2	0.627069	0.438596	¯0.877193
3	2	1.25414	¯0.561404	0.122807
4	3	0.0838504	1.55556	¯0.555556
5	3	0.0838504	¯0.444444	1.44444
6	3	1.17391	¯0.444444	¯0.555556

t = term number
p = posynomial
Ct = coefficient

Solution of Posynomial Subproblem

---------------Exponentiating LP Dual solution---------------
X = 3.76668 4.85262
Weights (ρ):
1
0.279595 0.720405
0.274237 0.455157 0.270606

Objective functions:
Primal: 3.76668 Dual: 3.76668
Duality Gap: 0 = 0 percent

Constraints:

Value	1.00384	1.0005
Infeasibility	0.00384363	0.000499041
Lambda	3.39035	1.03847

Sigmonial Problem at x = 3.76668 4.85262

Values of terms (Ut)

t	p	Ut
1	1	3.76668
2	2	0.941669
3	2	2.42631
4	2	¯0.886741
5	2	¯1.47175
6	3	0.487292
7	3	0.80877
8	3	0.48084
9	3	¯0.776215

Objective function = 3.76668
Constraint function values: 1.00949 1.00069
Infeasibilities: 0.00949319 0.000686784

Weights of negative terms, used for condensation:
ρ= 0.26403 0.438217 0.437005
|Δρ| = 0.0166718 0.000379617 0.00743935

Condensation of Sigmonial GP at x= 3.76668 4.85262

Number of variables: 2
Number of posynomials: 3
Total number of terms: 6
Degrees of difficulty: 3
Terms per posynomial: 1 2 3

Coefficients and exponent matrix:

t	p	Ct	exponents	
1	1	1	1	0
2	2	0.598618	0.47194	¯0.876434
3	2	1.19724	¯0.52806	0.123566
4	3	0.0839991	1.56299	¯0.562995
5	3	0.0839991	¯0.437005	1.43701
6	3	1.17599	¯0.437005	¯0.562995

t = term number
p = posynomial
Ct = coefficient

Solution of Posynomial Subproblem

---------------Exponentiating LP Dual solution---------------
X = 3.75146 4.85879
Weights (ρ):
1
0.278524 0.721476
0.272312 0.456797 0.270891

Objective functions:
Primal: 3.75146 Dual: 3.75146
Duality Gap: 0 = 0 percent

Constraints:

Value	1.00364	1.00039
Infeasibility	0.00363975	0.00038668
Lambda	3.8615	1.19831

Sigmonial Problem at x = 3.75146 4.85879

Values of terms (Ut)

t	p	Ut
1	1	3.75146
2	2	0.937864
3	2	2.4294
4	2	¯0.87959
5	2	¯1.47549
6	3	0.482748
7	3	0.809799
8	3	0.480229
9	3	¯0.772097

Objective function = 3.75146
Constraint function values: 1.01218 1.00068
Infeasibilities: 0.0121795 0.000679065

Weights of negative terms, used for condensation:
ρ= 0.262166 0.439778 0.435697
|Δρ| = 0.0018635 0.00156129 0.00130838

Condensation of Signomial GP at x= 3.75146 4.85879

Number of variables: 2
Number of posynomials: 3
Total number of terms: 6
Degrees of difficulty: 3
Terms per posynomial: 1 2 3

Coefficients and exponent matrix:

t	p	Ct	exponents	
1	1	1	1	0
2	2	0.598617	0.475667	¯0.879556
3	2	1.19723	¯0.524333	0.120444
4	3	0.0840273	1.5643	¯0.564303
5	3	0.0840273	¯0.435697	1.4357
6	3	1.17638	¯0.435697	¯0.564303

t = term number
p = posynomial
Ct = coefficient

Solution of Posynomial Subproblem

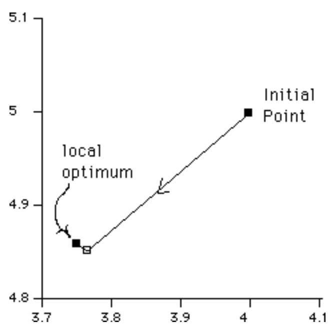
---------------Exponentiating LP Dual solution--------------

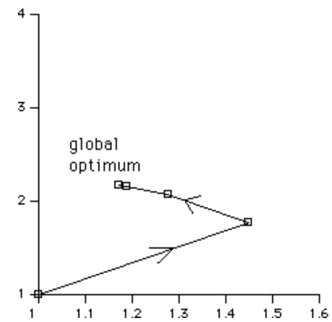
X = 3.7513 4.85889
Weights (ρ):
1
0.278512 0.721488
0.27229 0.456817 0.270892

Objective functions:
Primal: 3.7513 Dual: 3.7513
Duality Gap: 0 = 0 percent

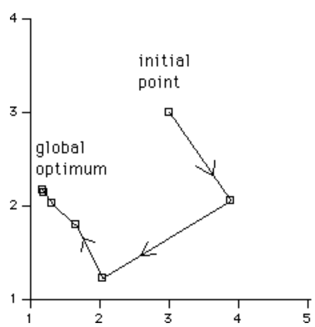
Constraints:

Value	1.00364	1.00039
Infeasibility	0.00363743	0.000385446
Lambda	3.93748	1.25877

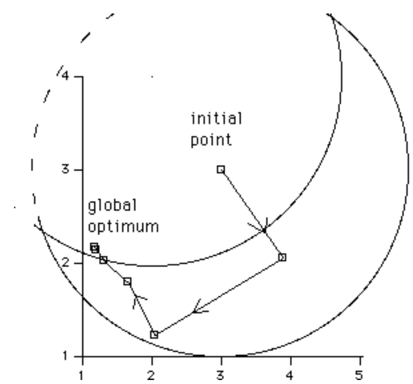




Starting at point (1,1)



Starting at point (3,3)



Starting at point (3,3)