

### Eliminating sets from consideration

Suppose that  $P_j$  is a lower bound on the cost of all feasible solutions which include set  $j$ .

If  $P_j$  exceeds the cost of a known feasible solution, then set  $j$  can be eliminated from any further consideration... it cannot be included in any optimal solution.

*How can we compute such a lower bound?*



If we add the restriction  $X_j=1$  to the set covering problem, then the optimal solution of the Lagrangian relaxation (with the added restriction) will be

$$P_j = \begin{cases} \Phi(\lambda) + \bar{C}_j & \text{if } X_j=0 \text{ in the solution of the current Lagrangian relaxation} \\ \Phi(\lambda) & \text{if } X_j=1 \text{ in the solution of the current Lagrangian relaxation} \end{cases}$$

*reduced cost of  $X_j$*

This gives us a lower bound on the cost of all feasible solutions of the set covering problem which contain set  $j$ .

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If any points remain uncovered, our heuristic adjustment procedure must

- select the next point to be covered
- select one of the sets covering this point

After all points are covered, any superfluous set (whose removal does not leave any point uncovered) is removed from the solution.

Consider a modification of our original problem, in which the constraint  $X_j=1$  is added.

Consider further the Lagrangian relaxation of that problem (in which all but the constraint  $X_j=1$  is relaxed).

If we use the same values of the Lagrangian multipliers as those used in the relaxation of the original problem, the Lagrangian relaxation of the modified problem is easily solved:

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### Heuristic Adjustment Procedures

Each time we evaluate  $\Phi(\lambda)$ , i.e., solve a Lagrangian relaxation, we obtain a solution which in general leaves some points uncovered and/or covers some points more than once.

By adding &/or removing sets, we can perhaps obtain a good solution of the original problem.

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### Selecting an uncovered point

- select one of the uncovered points at random
- select the uncovered point having the largest Lagrangian multiplier
- select the point having the smallest number of potential covering sets

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### Selecting a set to be added:

- Add the set which covers the point at lowest cost (Beasley)
- Consider as candidates the  $k$  least-cost sets covering the point; add the set which has the lowest "reduced cost".
- Consider as candidates the  $k$  least-cost sets covering the point; recompute the "reduced cost" with zero assigned to Lagrangian multipliers of points already covered.

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