Giapetto Toys

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Giapetto, Inc. manufactures and sells two wooden products:

- sets of toy soldiers
- toy trains,

using only two resources:

- lumber
- labor.

The toys can be made from either

- Grade **A** lumber, *or* (allowing for scrap)
- Grade **B** lumber:

Resource \ product	Soldier Set	Train
Grade A Lumber	3 board feet	5 board feet
Grade B Lumber	4 board feet	8 board feet
Labor	2 hours	4 hours

90,000 hours of labor will be available for production.

A shipment of 150,000 board feet of lumber will be received before production begins.

The quality of the lumber is uncertain, however, and will not be determined until *after* the production is scheduled.

Based upon past experience with the supplier, the following cases with their probabilities have been identified:

Case:	1	2	3
Probability	25%	50%	25%
Grade A	125,000 bd ft	100,000 bd ft	75,000 bd ft
Grade B	25,000 bd ft	50,000 bd ft	75,000 bd ft

Demand for the products is also uncertain, with two cases having been identified:

Case:	1	2
Probability	40%	60%
Sets of soldiers	40,000	50,000
Toy trains	60,000	80,000

The revenue from sale of toy trains is \$50 and that from a set of toy soldiers is \$40.

Production quantities of the two products must be fixed *before* the lumber arrives and *before* the levels of demand are known.

After the lumber quality has been determined and the demand experienced, the company has the following *recourses* available:

- Buy sets of toy soldiers from another supplier at $35 each
- Buy trains at $45 each from another supplier
- Schedule overtime at an additional cost of $10/hour

The six scenarios which can occur, with their probabilities.

Scenario #	*Probability*	*Lumber quality*	*Demand*
1			
2			
3			
4			
5			
6			

b. Compute the expected value of the random elements of the problem, and solve the LP to determine the optimal production quantities if the expected values were to occur:

# sets of toy soldiers	*# wooden trains*

What is the optimal value of the LP? $___________

c. Using the production quantities found in (b), compute the optimal *recourses* for each scenario, the profit for each scenario.

Scenario	# soldier sets purchased	# trains purchased	overtime labor	profit
1				
2				
3				
4				
5				
6				

Expected profit: $__________

d. Find the production quantities which maximize the expected profits by solving the "deterministic equivalent" LP.

- What are the production quantities?

# sets of toy soldiers	*# wooden trains*

- What are the dimensions of the LP tableau? ______ × ______
- What is the maximum expected profit? $__________
- What is the recourse if the lumber is of the lowest quality (case #1) and the demand is the higher estimate (case #2)?

Scenario	*# soldier sets purchased*	*# trains purchased*

- What is the *value of the stochastic solution (VSS)*, i.e., the difference in expected profit when using the stochastic LP solution compared to the LP using the expected values? $________

Model definition:

Decision variables

- **First Stage:**
  - X_1 = # of sets of soldiers to be produced
  - X_2 = # toy trains to be produced
- **Second Stage (Recourse):**
  - Y_{1A} & Y_{1B} = # of sets of soldiers to be produced from Grade A and Grade B lumber, respectively
  - Y_{2A} & Y_{2B} = # of toy trains to be produced from Grade A and Grade B lumber, respectively
  - Z_1 & Z_2 = # of sets of soldiers and trains, respectively, to be purchased from outside source
  - S_1 & S_2 = # of sets of soldiers and trains, respectively, sold
  - T = # hours of overtime used

Second Stage Optimization Problem:

After X has selected in the first stage, and we have observed the random values of

- A & B (the quantities of Grades A & B lumber, respectively), and
- D_1 and D_2 (the demands for toy soldiers and trains, respectively).

Find the maximum revenue minus costs:

$$Q(X) = Maximum \quad 40S_1 + 50S_2 - 35Z_1 - 45Z_2 - 10T$$

subject to

Production schedule is fulfilled: $\begin{cases} Y_{1A} + Y_{1B} = X_1 \\ Y_{2A} + Y_{2B} = X_2 \end{cases}$

Lumber resource constraints:
where A & B are the random available quantities of Grades A & B lumber, in board feet.

Labor resource constraint: $2X_1 + 4X_2 - T = 90000$

Sales consist of produced & purchased products: $\begin{cases} S_1 = Y_{1A} + Y_{1B} + Z_1 \\ S_2 = Y_{2A} + Y_{2B} + Z_2 \end{cases}$

Sales limited by demand: $\begin{cases} S_1 \le D_1 \\ S_2 \le D_2 \end{cases}$

where D_1 & D_2 are the random demands for the two products.

nonnegativity: $Y_{1A}, Y_{1B}, Y_{2A}, Y_{2B}, S_1, S_2, Z_1, Z_2, \& \; T \ge 0$

First-stage Optimization Problem:

$$\underset{X \ge 0}{Maximize} \quad \sum_{k \in K} p_k Q_k(X)$$

where K is the set of scenarios, and

p_k the probability of scenario #k.

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First-stage data:
 A,B=
1 1 > 0

 i   variable  cost
  1  X[1]        0
  2  X[2]        0
  Objective:  Minimize
-------------------------
Second-stage data
K= # scenarios = 6
 The following data vary
by scenario:   h
Costs:
i var.  q
1 Y1A   0
2 Y1B   0
3 Y2A   0
4 Y2B   0
5 Z1   35
6 Z2   45
7 S1  ¯40
8 S2  ¯50
9 T    10
```

```
Technology matrix T
(coefficients of
X in 2nd stage) =
 ¯1  0
  0 ¯1
  0  0
  0  0
  2  4
  0  0
  0  0
  0  0
  0  0

Technology matrix W (coefficients of
Y in 2nd stage) =
 1 1 0 0 0 0  0  0  0
 0 0 1 1 0 0  0  0  0
 3 0 5 0 0 0  0  0  0
 0 4 0 8 0 0  0  0  0
 0 0 0 0 0 0  0  0 ¯1
 1 1 0 0 1 0 ¯1  0  0
 0 0 1 1 0 1  0 ¯1  0
 0 0 0 0 0 0  1  0  0
 0 0 0 0 0 0  0  1  0
```

```
Right-hand-sides in second stage =
 k p[k] 1 2    3     4     5    6 7    8     9
```

```
 _ ____ _ _ ______ _____ _____ _ _ _____ _____

 1 0.1  0 0 125000 25000 90000 0 0 40000 60000
 2 0.2  0 0 100000 50000 90000 0 0 40000 60000
 3 0.1  0 0  75000 75000 90000 0 0 40000 60000
 4 0.15 0 0 125000 25000 90000 0 0 50000 80000
 5 0.3  0 0 100000 50000 90000 0 0 50000 80000
 6 0.15 0 0  75000 75000 90000 0 0 50000 80000
-----------------------------------------------
Note:  h varies   by scenario
=============================================================

Optimal Solution
 (Found by solving deterministic
equivalent
problem directly, without
decomposition)
Total objective function: ¯2091250

Stage One: nonzero  variables:
 i   variable  value

  1  X[1]      40000
  2  X[2]       2250
  3  surplus_1 42250
-------------------------------
Second Stage: nonzero variables

Scenario #1
i   variable  value
  1  Y1A       38250
  2  Y1B        1750
  4  Y2B        2250
  6  Z2        57750
  7  S1        40000
  8  S2        60000
 10  slack_3   10250
 12  slack_5    1000
-------------------------------
Scenario #2

 i   variable  value
  1  Y1A       32000
  2  Y1B        8000
  4  Y2B        2250
  6  Z2        57750
  7  S1        40000
```



```
  8  S2        60000
 10  slack_3    4000
 12  slack_5    1000
-------------------------------
Scenario #3
 i   variable  value
  1  Y1A       21250
  2  Y1B       18750
  3  Y2A        2250
  6  Z2        57750
  7  S1        40000
  8  S2        60000
 12  slack_5    1000
-------------------------------
Scenario #4
 i   variable  value
  1  Y1A       38250
  2  Y1B        1750
  4  Y2B        2250
  5  Z1        10000
  6  Z2        77750
  7  S1        50000
  8  S2        80000
 10  slack_3   10250
 12  slack_5    1000
-----------------------------
Scenario #5
 i   variable  value
  1  Y1A       32000
  2  Y1B        8000
  4  Y2B        2250
  5  Z1        10000
  6  Z2        77750
  7  S1        50000
  8  S2        80000
 10  slack_3    4000
 12  slack_5    1000
-----------------------------
Scenario #6
 i   variable  value
  1  Y1A       21250
  2  Y1B       18750
  3  Y2A        2250
  5  Z1        10000
  6  Z2        77750
  7  S1        50000
  8  S2        80000
 12  slack_5    1000
```

Solution with Perfect Information

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Solution for scenario #1

Optimal cost: ¯2023750
Stage One: nonzero variables:
 i       value     name
```



```
 1    40000.00 X[1]
 2     4750.00 X[2]
 3    44750.00 surplus_1

Second-stage: nonzero variables
 i       value  name
 1    33750.00 Y1A
 2     6250.00 Y1B
 3     4750.00 Y2A
 6    55250.00 Z2
 7    40000.00 S1
 8    60000.00 S2
 9     9000.00 T
------------------------------
Solution for scenario #2

Optimal cost: ¯2017500
Stage One: nonzero variables:
 i       value     name

 1    40000.00 X[1]
 2     3500.00 X[2]
 3    43500.00 surplus_1

Second-stage: nonzero variables
 i       value  name
 1    27500.00 Y1A
 2    12500.00 Y1B
 3     3500.00 Y2A
 6    56500.00 Z2
 7    40000.00 S1
 8    60000.00 S2
 9     4000.00 T

Solution for scenario #3

Optimal cost: ¯2001250
Stage One: nonzero variables:
 i       value     name
 1    40000.00 X[1]
 2     2250.00 X[2]
 3    42250.00 surplus_1

Second-stage: nonzero variables
 i       value    name
 1    21250.00 Y1A
 2    18750.00 Y1B
 3     2250.00 Y2A
 6    57750.00 Z2
 7    40000.00 S1
 8    60000.00 S2
12     1000.00 slack_5

Solution for scenario #4

Optimal cost: ¯2268750
Stage One: nonzero variables:
 i       value     name
```



```
 1    47916.67 X[1]
 3    47916.67 surplus_1
Second-stage: nonzero variables
 i       value  name
 1    41666.67 Y1A
 2     6250.00 Y1B
 5     2083.33 Z1
 6    80000.00 Z2
 7    50000.00 S1
 8    80000.00 S2
 9     5833.33 T

Solution for scenario #5

Optimal cost: ¯2237500
Stage One: nonzero variables:
 i       value     name
 1    45833.33 X[1]
 3    45833.33 surplus_1


Second-stage: nonzero variables
 i       value  name
 1    33333.33 Y1A
 2    12500.00 Y1B
 5     4166.67 Z1
 6    80000.00 Z2
 7    50000.00 S1
 8    80000.00 S2
 9     1666.67 T

Solution for scenario #6

Optimal cost: ¯2181250
Stage One: nonzero variables:
 i       value     name
 1    43750.00 X[1]
 3    43750.00 surplus_1

Second-stage: nonzero variables
 i       value    name
 1    25000.00 Y1A
 2    18750.00 Y1B
 5     6250.00 Z1
 6    80000.00 Z2
 7    50000.00 S1
 8    80000.00 S2
12     2500.00 slack_5

Expected cost with perfect
information: ¯2144750
```

Certainty-Equivalent Tableau

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b	z	1	2	3	1	2	3	4	5	6	7	8	9	0	1	2	3	4
0	1	0	0	0	0	0	0	0	35	45	¯40	¯50	10	0	0	0	0	0
0	0	1	1	¯1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	¯1	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	¯1	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0
100000	0	0	0	0	3	0	5	0	0	0	0	0	0	1	0	0	0	0
50000	0	0	0	0	0	4	0	8	0	0	0	0	0	0	1	0	0	0
90000	0	2	4	0	0	0	0	0	0	0	0	0	¯1	0	0	1	0	0
0	0	0	0	0	1	1	0	0	1	0	¯1	0	0	0	0	0	0	0
0	0	0	0	0	0	0	1	1	0	1	0	¯1	0	0	0	0	0	0
46000	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	0
72000	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1

Optimal Solution
Found by solving certainty equivalent problem,
i.e., replacing all random parameters by their expected values.

Total objective function: ¯2177500
Stage One: nonzero variables:

i	variable	value
1	X[1]	45833.33333
3	surplus_1	45833.33333

Second Stage: nonzero variables

i	variable	value
1	Y1A	33333.3333333
2	Y1B	12500.0000000
5	Z1	166.6666667
6	Z2	72000.0000000
7	S1	46000.0000000
8	S2	72000.0000000
9	T	1666.6666667