RELAXATIONS

Consider a typical integer programming problem:

Problem (P):

$$\text{Find } z' = \min\left\{ c(x) \,\middle|\, x \in X \subseteq Z^n \right\}$$

where

Z is the set of nonnegative integers $\{0,1,2,3,\ldots\}$

Z^n is the set of n-dimensional vectors of nonnegative integers

Problem P': Find $z = \min\left\{ f(x) \,\middle|\, x \in X' \subseteq R^n \right\}$

is a ***relaxation*** of problem P if:

1. the feasible region X' of P' contains the feasible region of P, i.e., $X \subseteq X'$
2. the objective value in P' is no worse than that of P for all x in the domain of P, i.e., c(x) ≥ f(x) for all x in X *(for minimization).*

Common relaxations:

- Dropping integer restrictions (the *LP relaxation* of a (linear) MIP);
- Dropping some constraints.
- Aggregating constraints (*surrogate constraint*);
- Lagrangian relaxation
- Replacing cost function by linear underestimate

Propositions:

- If P' is a relaxation of P, then z' ≤ z *(in case of minimization)*
- If P' is infeasible, then P is infeasible
- If x* solves P' and is feasible in P (i.e., x*∈ X), and f(x*)=c(x*), then x* solves P

These propositions imply that a relaxation can be used to fathom nodes of a search tree (*branch-&-bound method*).