

# Russell-Yasuda Kasai Model

## Multi-stage

### Asset / Liability Management

- Cariño, D. R., T. Kent, et al. (1994). "The Russel-Yasuda Kasai model: An asset-liability model for a Japanese insurance company using multistage stochastic programming." *Interfaces* **24**(1): 29-49.
- Cariño, D. R., D. H. Myers, et al. (1998). "Concepts, technical issues, and uses of the Russell-Yasuda Kasai financial planning model." *Operations Research* **46**(4): 450-462.
- Cariño, D. R. and W. T. Ziemba (1998). "Formulation of the Russell-Yasuda Kasai financial planning model." *Operations Research* **46**(4): 433-449.

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#### Random Variables

$rp_{jt}$  = price return of asset  $j$  in period  $t$   
 $ri_{jt}$  = income return of asset  $j$  in period  $t$   
 $F_t$  = deposit inflow in period  $t$   
 $P_t$  = principal payout in period  $t$   
 $I_t$  = income payout in period  $t$   
 $g_t$  = rate of interest paid on policies in period  $t$   
 $L_t$  = liability valuation at end of period  $t$

#### Decision Variables

$w_{jt}$  = market value held in asset  $j$  in period  $t$   
 $W_t$  = total fund market value in period  $t$   
 $u_t$  = income shortfall in period  $t$   
 $v_t$  = income surplus in period  $t$

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#### Constraints include:

$$\begin{aligned}
 \sum_{j=1}^J w_{jt} &= W_t \\
 W_{t+1} - \sum_{j=1}^J (1 + rp_{j,t+1} + ri_{j,t+1}) w_{jt} &= F_{t+1} - P_{t+1} - I_{t+1} \\
 \sum_{j=1}^J ri_{j,t+1} w_{jt} + u_{t+1} - v_{t+1} &= g_{t+1} L_t \\
 w_{jt} \geq 0, \quad u_t \geq 0, \quad v_t \geq 0
 \end{aligned}$$

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#### Size of Problem (# asset classes=7)

# rows: 263

# columns: 431

#### Size of Deterministic Equivalent Problem

# rows ~ 31,000

# columns ~ 44,000

Nested Benders' Decomposition was used.

Three hours of computation was required to solve full model.

Contributed 79 million US\$ in first two years of use.

Developed by Frank Russell Company and The Yasuda Fire & Marine Insurance Company.

Decisions are made on how best to invest in assets to meet a random liability stream over time, with random investment returns.

Goal is to produce a high-income return to pay annual interest on savings-type insurance policies while maximizing the long-term wealth of the firm.

Handles complex regulations imposed by Japanese insurance laws and practices.

Model is multistage stochastic LP with recourse

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#### Objective:

$$\text{Maximize } E \left[ W_H - \sum_{t=1}^H c_t(u_t) \right]$$

where  $c_t(u_t)$  is a piecewise-linear concave function which specifies the penalties for income shortfalls.

(Converts to LP with introduction of additional variables.)



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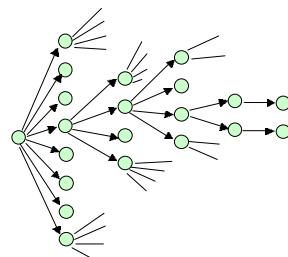
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Periods are of varying length:

8 branches in period 1 (first quarter)  
 4 branches in period 2 (remainder of first year)  
 4 branches in period 3 (year 2)  
 2 branches in period 4 (years 3-5)  
 1 branch in period 6 (terminal conditions)

Total number of scenarios:  $8 \times 4 \times 4 \times 2 \times 1 = 256$



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