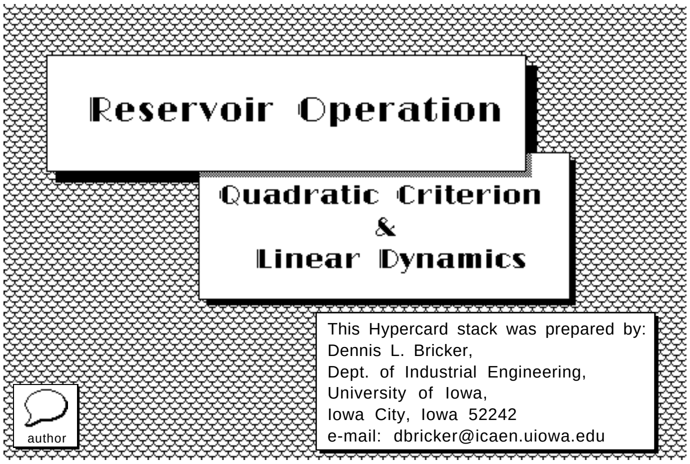

Reservoir Operation

Quadratic Criterion & Linear Dynamics

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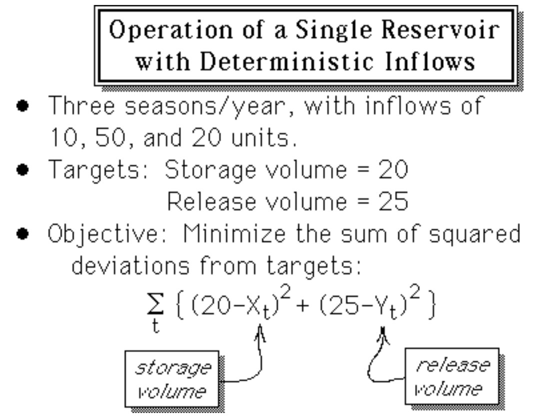
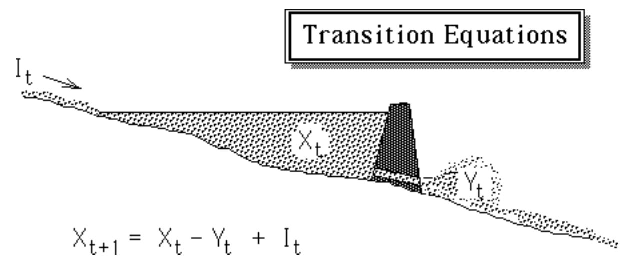


Operation of a Single Reservoir with Deterministic Inflows

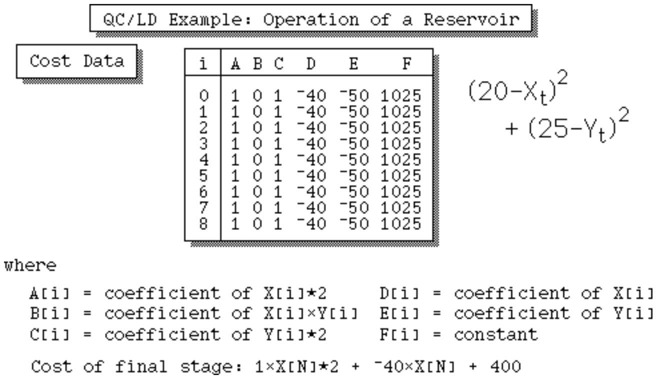
- Three seasons/year, with inflows of 10, 50, and 20 units.
- Targets: Storage volume = 20
  Release volume = 25
- Objective: Minimize the sum of squared deviations from targets:

$$\sum_t \{ (20-X_t)^2 + (25-Y_t)^2 \}$$



Transition Equations

$$X_{t+1} = X_t - Y_t + I_t$$



QC/LD Example: Operation of a Reservoir

Cost Data

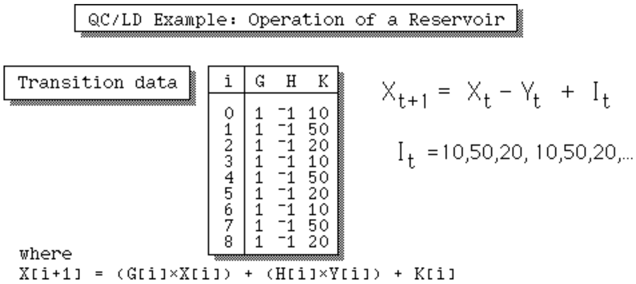
i	A	B	C	D	E	F
0	1	0	1	¯40	¯50	1025
1	1	0	1	¯40	¯50	1025
2	1	0	1	¯40	¯50	1025
3	1	0	1	¯40	¯50	1025
4	1	0	1	¯40	¯50	1025
5	1	0	1	¯40	¯50	1025
6	1	0	1	¯40	¯50	1025
7	1	0	1	¯40	¯50	1025
8	1	0	1	¯40	¯50	1025

$$(20-X_t)^2 + (25-Y_t)^2$$

where

```
A[i] = coefficient of X[i]*2      D[i] = coefficient of X[i]
B[i] = coefficient of X[i]×Y[i]   E[i] = coefficient of Y[i]
C[i] = coefficient of Y[i]*2      F[i] = constant
```

Cost of final stage: 1×X[N]*2 + ¯40×X[N] + 400



QC/LD Example: Operation of a Reservoir

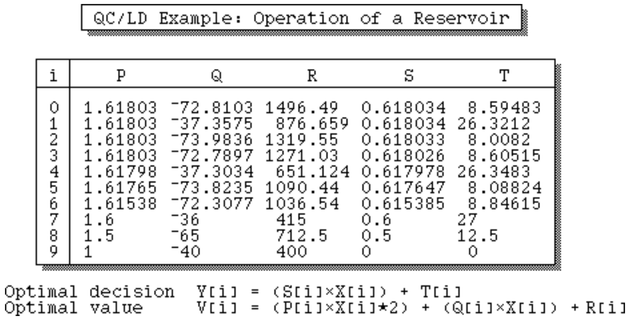
Transition data

i	G	H	K
0	1	¯1	10
1	1	¯1	50
2	1	¯1	20
3	1	¯1	10
4	1	¯1	50
5	1	¯1	20
6	1	¯1	10
7	1	¯1	50
8	1	¯1	20

$$X_{t+1} = X_t - Y_t + I_t$$

I_t =10,50,20, 10,50,20,...

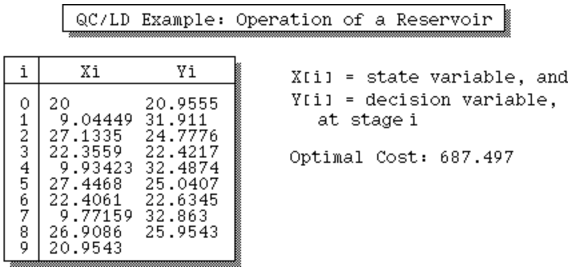
where
X[i+1] = (G[i]×X[i]) + (H[i]×Y[i]) + K[i]



QC/LD Example: Operation of a Reservoir

i	P	Q	R	S	T
0	1.61803	¯72.8103	1496.49	0.618034	8.59483
1	1.61803	¯37.3575	876.659	0.618034	26.3212
2	1.61803	¯73.9836	1319.55	0.618033	8.0082
3	1.61803	¯72.7897	1271.03	0.618026	8.60515
4	1.61798	¯37.3034	651.124	0.617978	26.3483
5	1.61765	¯73.8235	1090.44	0.617647	8.08824
6	1.61538	¯72.3077	1036.54	0.615385	8.84615
7	1.6	¯36	415	0.6	27
8	1.5	¯65	712.5	0.5	12.5
9	1	¯40	400	0	0

```
Optimal decision  Y[i] = (S[i]×X[i]) + T[i]
Optimal value     V[i] = (P[i]×X[i]*2) + (Q[i]×X[i]) + R[i]
```

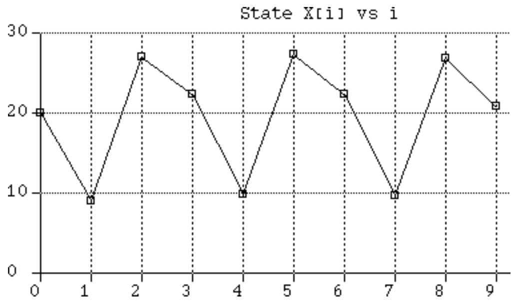


QC/LD Example: Operation of a Reservoir

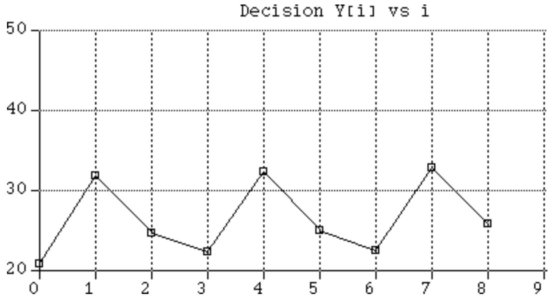
i	Xi	Yi
0	20	20.9555
1	9.04449	31.911
2	27.1335	24.7776
3	22.3559	22.4217
4	9.93423	32.4874
5	27.4468	25.0407
6	22.4061	22.6345
7	9.77159	32.863
8	26.9086	25.9543
9	20.9543	

X[i] = state variable, and
Y[i] = decision variable,
at stage i

Optimal Cost: 687.497



State X[i] vs i



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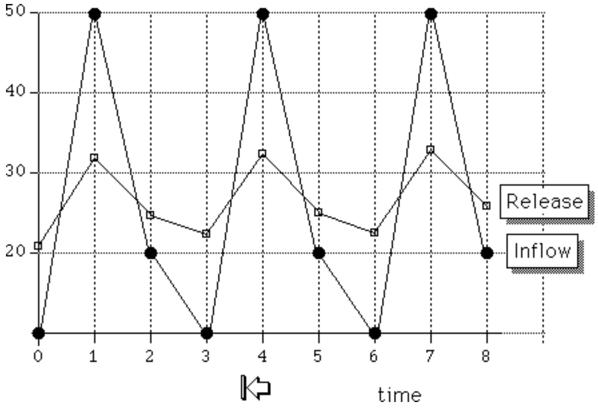

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