The Peter Principle of Industrial Mobility

The theory that employees within an organization will advance to their highest level of competence and then be promoted to and remain at a level at which they are incompetent.

-- *American Heritage® Dictionary of the English Language*

Coined by Laurence Johnston *Peter* (1919–1990).

That is, everyone rises to their level of incompetence.

The draftsman position at a large engineering firm can be occupied by a worker at any of three levels:

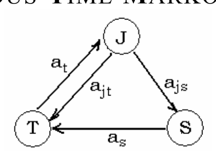
T= Trainee
J = Junior Draftsman
S = Senior Draftsman

Assume that a Trainee stays at a rank for an exponentially-distributed length of time (with parameter $\mathbf{a_t}$) before being promoted to Junior Draftsman.

A Junior Draftsman stays at that level for an exponentially-distributed length of time (with parameter $\mathbf{a_j} = \mathbf{a_{jt}} + \mathbf{a_{js}}$). Then he either leaves the position and is replaced by a Trainee (with probability a_{jt}/a_j), or is promoted to a Senior Draftsman (with probability a_{js}/a_j).

Senior Draftsmen remain in that position an exponentially-distributed length of time (with parameter $\mathbf{a_s}$) before resigning or retiring, in which case they are replaced by a Trainee.

CONTINUOUS-TIME MARKOV MODEL



The rank of a person in a draftsman's position may be modeled as a continuous-time Markov chain with transition rate matrix:

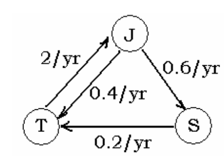
$$\mathbf{A} = \begin{array}{c|ccc} & T & J & S \\ \hline T & -a_t & a_t & 0 \\ J & a_{jt} & -a_j & a_{js} \\ S & a_s & 0 & -a_s \end{array}$$

For example, suppose that the mean time in the three ranks are:

State	Mean Time
T	.5 years
J	1 year
S	5 years

and that a Junior Draftsman

- leaves and is replaced by a Trainee with probability 40%
- is promoted with probability 60%.



Then the *transition rate matrix* is

$$\Lambda = \begin{bmatrix} -2 & 2 & 0 \\ 0.4 & -1 & 0.6 \\ 0.2 & 0 & -0.2 \end{bmatrix}$$

The *steady-state distribution* is computed by solving

$$\pi\Lambda = 0 \Rightarrow \begin{cases} -2\pi_1 + 0.4\pi_2 + 0.2\pi_3 = 0 \\ 2\pi_1 - \pi_2 = 0 \\ 0.6\pi_2 - 0.2\pi_3 = 0 \end{cases}$$

and

$$\sum_i \pi_i = 1$$

which has the solution:

$\pi_t = 0.11$
$\pi_j = 0.22$
$\pi_s = 0.67$

That is, 11% of the workforce will be trainees, 22% junior draftsmen, etc.

The "Peter Principle"

The duration that people spend in any given rank is *not* exponentially distributed in general.
A *bimodal distribution* is often observed in which many people leave (are promoted) rather quickly, while others persist for a substantial time.

The "Peter Principle" asserts that a worker is promoted until first reaching a position in which he or she is incompetent.
When this happens, the worker stays in that job until retirement.

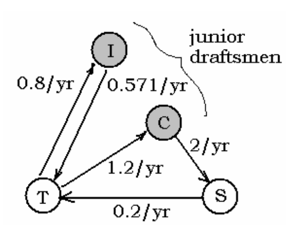
Let's modify the above model by classifying 60% of the Junior Draftsmen:

- 60% are **Competent**
- 40% are **Incompetent,**

represented by states **C** and **I**, respectively.

Suppose that *Incompetent* junior draftsmen stay at that rank until quitting or retirement (after an average of 1.75 years), and
Competent junior draftsmen are promoted (after an average of 0.5 years),
These values have been chosen so that the average time spent in the rank of junior draftsman is *still*

$(0.6)(.5) + (0.4)(1.75) = 1$ year (as before)



The transition rate matrix is now

	T	I	C	S
T	–2	0.8	1.2	0
I	0.571	–.571	0	0
C	0	0	–2	2
S	0.2	0	0	–0.2

The *steady-state distribution* is now:

$\pi_t = 0.111$
$\pi_i = 0.155$
$\pi_c = 0.067$
$\pi_s = 0.667$

•Note first that π_t and π_s agree with the previous results,
and that π_j computed earlier equals $\pi_i + \pi_c$.

•Secondly, note that while only 40% of the Junior Draftsmen are incompetent,
in steady state a person holding the rank of Junior Draftsman is found to be incompetent with probability $\pi_i/(\pi_i+\pi_c) = 70\%$!