Multistage Manufacturing System with Product Inspection, Rejection & Rework

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A machined part requires the following sequence of steps:

- Machine A
- Inspection A
- Machine B
- Inspection B
- Machine C
- Inspection C
- Pack & Ship

During each machining step, parts could be ruined (e.g., because of a casting defect).

In the inspection step following each machine, the part may be either

- passed to the next stage
- rejected and scrapped
- returned to the preceding machine for rework

Example Data

Cost of blank part: $50
Salvage value of scrapped part: $12

Operation	*Time Rqmt (hr)*	*Operating Cost ($/hr)*	*Scrap Rate (%)*	*% Sent Back for Rework*
Machine A	5.0	12.00	15	
Inspect A	1.6	10.00	5	7
Machine B	3.0	12.00	6	
Inspect B	1.6	10.00	4	4
Machine C	2.7	15.00	5	
Inspect C	1.6	10.00	8	8
Pack & Ship	0.7	5.00		

- How many blank parts are expected to obtain each successfully-completed part?
- What is the estimated cost to obtain each successfully-completed part?

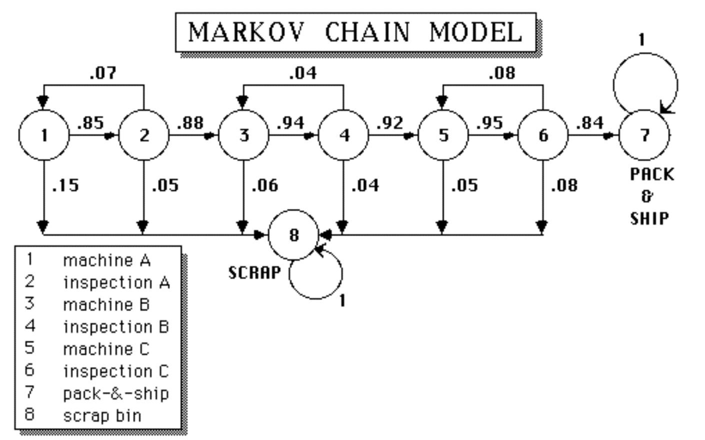
Discrete-Time Markov Chain Model

Define a stochastic process $\{X_i\}$ describing a part, where

X_i = current process after i transitions, where

State	*Location of Part*
1	Machine A
2	Inspection A
3	Machine B
4	Inspection B
5	Machine C
6	Inspection C
7	Packing-&-Shipping Department
8	Scrap Bin

Note that this is a discrete-(time)parameter process, but the length of each stage is not of fixed duration!

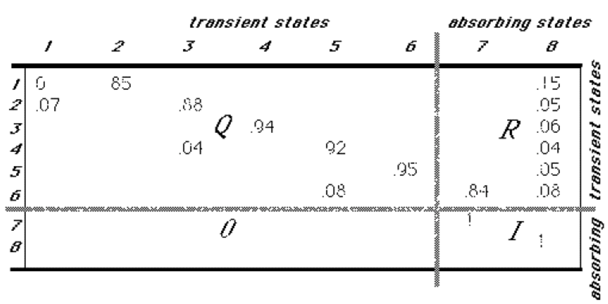


Note that states **7** & **8** are **absorbing**!

Transition Probability Matrix

	1	2	3	4	5	6	7	8
1		0.85						0.15
2	0.07		0.88					0.05
3				0.94				0.06
4			0.04		0.92			0.04
5						0.95		0.05
6					0.08		0.84	0.08
7							1	
8								1

Partition of the Matrix:



$$P = \begin{bmatrix} Q & R \\ 0 & I \end{bmatrix}$$

Q=transient-to-transient transition probabilities
R=transient-to-absorbing transition probabilities

$E = (I - Q)^{-1}$ = Expected Number of Visits to Transient States

	1	2	3	4	5	6
1:	1.06	.904	.826	.777	.773	.735
2:	.074	1.06	.972	.914	.91	.864
3:	0	0	1.04	.977	.972	.924
4:	0	0	.042	1.04	1.03	.983
5:	0	0	0	0	1.08	1.03
6:	0	0	0	0	.087	1.08

For example, a part currently residing in state 3 (Machine B) is expected to visit Machine B **1.04** times (including the current visit), Machine C **0.972** times, etc.

$A = ER = (I - Q)^{-1} R$ = Absorption Probabilities

	7	8
1:	.617	.383
2:	.726	.274
3:	.776	.224
4:	.826	.174
5:	.864	.136
6:	.909	.091

For example, a part currently residing in state 3 (Machine B) has probability **22.4%** that it will eventually reach state 8, the "scrap bin".

Likewise, a part which enters the system in state 1 has **61.7%** probability that it will eventually reach state 7, the "Pack-&-Ship" department.

Expected Man-hour Requirements per Entering Part

OPERATION	STATE	MAN-HR / ENTERING PART
MACHINE A	1	5.0 × 1.06 = 5.300
INSPECTION A	2	1.6 × .904 = 1.446
MACHINE B	3	3.0 × .826 = 2.478
INSPECTION B	4	1.6 × .777 = 1.243
MACHINE C	5	2.7 × .773 = 2.087
INSPECTION C	6	1.6 × .735 = 1.176
PACK & SHIP	7	0.7 × .617 = 0.432
		TOTAL = 14.162 man-hrs

hrs/visit x # visits

To obtain one successfully completed part, we expect to use $\frac{1}{0.617}$=1.6207 entering parts.

Therefore, to obtain the man-hour requirements at each stage for a successfully-completed part, we multiply the hours/visit for each entering part by the factor **1.6207**.

For example, the total man-hour requirements for each completed part is 1.6207×14.162 = **22.95** man-hours.

Expected Direct Costs Per Completed Part

Materials: $50 × 1.6207 = **$81.04**

Scrap value recovered: $12 × 0.383 × 1.6207 = **$7.45**

OPERATIONS COST

OPERATION	HOURLY RATE	MAN-HRS	TOTAL COST
MACHINE A	12.00	8.613	103.40
INSPECTION A	10.00	2.343	23.43
MACHINE B	12.00	4.017	48.20
INSPECTION B	10.00	2.014	20.14
MACHINE C	15.00	3.383	50.75
INSPECTION C	10.00	1.905	19.05
PACK-&-SHIP	5.00	.700	3.50
		TOTAL =	$ 268.40

Total: $81.04 + $268.40 – $7.45 = **$341.99**

An Optimization Problem

- Suppose that we must decide upon a least-cost production lotsize X* in order to obtain 15 acceptable parts.
- There is a fixed **setup** cost of **$10000** for the machining processes. Assume no salvage value for unacceptable parts.
- If after the initial lot, fewer than 15 acceptable parts have been obtained (so that an additional ***n*** parts are required), then what should be the next lotsize X*(n)? That is, we want an optimal decision **policy** X*(n), n=1,2,...15.

Dynamic Programming Solution *(Click here for details).*

n	*X*(n)*	*p·X*(n)*	*Expected cost*
1	4	2.468	1082.53
2	6	3.702	1611.24
3	9	5.553	2095.73
4	11	6.787	2547.48
5	13	8.021	2987.49
6	14	8.638	3411.28
7	16	9.872	3826.39
8	18	11.106	4236.44
9	20	12.340	4643.35
10	22	13.574	5048.38
11	24	14.808	5452.40
12	25	15.425	5855.63
13	27	16.659	6243.10
14	29	17.893	6632.20
15	31	19.127	7022.99

That is, we should manufacture **31** parts, with an expected yield of 19.127 parts. The expected cost is **$7023** (not including the initial setup cost). If the actual yield is, for example, 13 parts, then an additional X*(2)=6 parts should be manufactured.