

## Machine Replacement Problem

At the beginning of each month, a machine is inspected and classified as:

- 1) Good as new
- 2) Operable, with minor deterioration
- 3) Operable, with major deterioration
- 4) Inoperable

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## Machine Replacement Problem

After determining the state of the machine, a decision must be made:

- 1) Keep the machine another month
- 2) Replace the machine with a new machine

# Machine Replacement Problem

A replacement machine costs \$3000, minus trade-in value:

\$1000 if in state 2

500 if in state 3

0 if in state 4

Monthly operating costs are \$100, \$200, and \$500 for a machine in states 1, 2, & 3, respectively.

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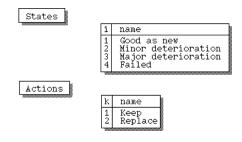
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# Machine Replacement Problem

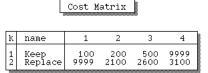
#### Survival probabilities

_ \to	:	State	;	
from: \	1	2	3	4
1 نه	0.75	0.1875	0.0625	0
State 5	_	0.75	0.1875	0.0625
$\sim$ 3		_	0.75	0.25

What is the optimal replacement policy?



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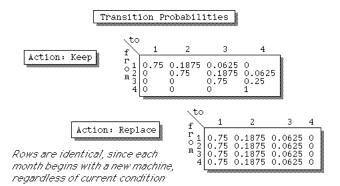


(Rows ~ actions, Columns ~ states)

 ${\tt A}$  value of 9999 above signals an infeasible action in a state.

(includes cost of operating the new machine, if decision is to replace)

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Linear Programming Approach

Policy Iteration Method

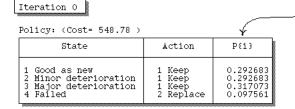
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					L	.P Table	au
k:	1	1	2	1	2	2	R
i:	1	2	2	3	3	4	H S
Min	100 0.25 -0.1875 -0.0625 1	0 0.25	-0.75 0.8125	0	-0.75 -0.1875		0 0 0 1

 $X_{i}^{k}$  = probability that machine is in state \*i and decision \*k is selected

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Steadystate distribution resulting from this policy



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Iteration 1

	**			*	*	
k:	1 1	2	1	2	2	
1:	1 2	2	3	3	4	rhs
Min	0 0 1 0 0 1 0 0 0 0	628.571 -1.71429 2.28571 0.285714 0.142857	111.607 0.428571 0.428571 0.366071 -0.223214	0 0 0 1 0	0 0 0 0	-513.393 0.428571 0.428571 0.116071 0.0267857

i~state, k~action

 $X_3^2$  has replaced  $X_3^1$  in the basis

# **Linear Programming Model**

What is the policy which minimizes the average cost/month in steady state?

Minimize 
$$\sum_{i=1}^{N} C_i^k X_i^k$$

where

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Iteration 0			In	ritial poli the maci	-	r: keep until ne fails
basic.	**		*		*	
k:	1 1	2	1	2	2	
_i:	1 2	2	3	3	4	rhs
Min	0 0 1 0 0 1 0 0 0 0	541.463 -2.04878 1.95122 0.780488 0.317073	0 0 0 1 0	-304.878 -1.17073 -1.17073 2.73171 0.609756	0 0 0 0 1	-548.78 0.292683 0.292683 0.317073 0.097561

i~state, k~action

Initial basis is obtained by selecting one basic variable for each state.

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Choose a column having negative reduced cost, and enter it into the basis:

basic	**		*		*	
k:	1 1	2	1	2	2	
i:	1 2	2	3	3	4	rhs
Min	0 0 1 0 0 1 0 0 0 0	541.463 -2.04878 1.95122 0.780488 0.317073	0 0 0 1 0	-304.878 -1.17073 -1.17073 -2.73171 0.609756	0 0 0 0 1	-548.78 0.292683 0.292683 0.317073 0.097561

choose pivot row, using minimum ratio test minimum  $\left\{ \frac{0.31707}{2.7317}, \frac{0.09756}{0.60975} \right\}$ =  $\frac{0.31707}{2.7317} = 0.116071$ 

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Iteration 1

Policy: (Cost= 513.393 )

State	Action	P{i}
1 Good as new	1 Keep	0.428571
2 Minor deterioration	1 Keep	0.428571
3 Major deterioration	2 Replace	0.116071
4 Failed	2 Replace	0.0267857

	**			*	*	
k:	1 1	2	1	2	2	
<u>i:</u>	1 2	2	3	3	4	rhs
Min	0 0 1 0 0 1 0 0 0 0	628.571 -1.71429 2.28571 0.285714 0.142857	111.607 0.428571 0.428571 0.366071 -0.223214	0 0 0 1 0	0 0 0 0	-513.393 0.428571 0.428571 0.116071 0.0267857

i~state, k~action

Reduced costs are nonnegative... the optimality condition is satisfied!

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# Policy Iteration Method

เ☞ Average cost per month

Present value of all future costs

40

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#### Machine Replacement Example

Policy Improvement Step: Evaluation of alternate actions

#### State #2, Minor deterioration

Current Policy: action #1, Keep g(R)+Vi(R) = -992.683

k	name	C'	ΔC
1 2	Keep Replace	-992.683 -451.22	0 541.463

no improvement can be achieved by changing action in this state.

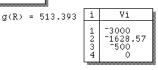
C'Ikl = cost if action k is selected for one stage  $\Delta \texttt{CIkl}$  = improvement (if <0)

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#### Machine Replacement Example

State	Action		
1 Good as new 2 Minor deterioration 3 Major deterioration 4 Failed	1 Keep 1 Keep 2 Replace 2 Replace	<b>T</b>	new policy

#### Value Determination



#### Optimal Policy

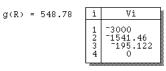
State	Action
1 Good as new	1 Keep
2 Minor deterioration	1 Keep
3 Major deterioration	2 Replace
4 Failed	2 Replace

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#### Machine Replacement Example

State	Action
1 Good as new	1 Keep
2 Minor deterioration	1 Keep
3 Major deterioration	1 Keep
4 Failed	2 Replace





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#### Machine Replacement Example

Policy Improvement Step: Evaluation of alternate actions

#### State #3, Major deterioration

Current Policy: action #1, Keep g(R)+Vi(R) = 353.659

k	name	C'	ΔC	
1 2	Keep	353.659	0	improve-
	Replace	48.7805	-304.878	ment

C'[k] = cost if action k is selected for one stage  $\Delta C[k]$  = improvement (if <0)

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#### Machine Replacement Example

Policy Improvement Step: Evaluation of alternate actions

#### State #2, Minor deterioration

Current Policy: action #1, Keep g(R)+Vi(R) = -1115.18

k	name	C'	ΔC
1 2	Keep	-1115.18	0
	Replace	-486.607	628.571

no improvement can be achieved by changing action in this state

C'[k] = cost if action k is selected for one stage  $\Delta$ C[k] = improvement (if <0)

#### Machine Replacement Example

Policy Improvement Step: Evaluation of alternate actions

#### State #3, Major deterioration

Current Policy: action #2, Replace g(R) + Vi(R) = 13.3929

k	name	C.	ΔC
1 2	Keep	125	111.607
	Replace	13.3929	0

no improvement can be achieved by changing action in this state.

C'[k] = cost if action k is selected for one stage  $\Delta C[k]$  = improvement (if <0)



The current policy is optimal!

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Let's begin with an initial policy: keep machine until it fails i.e., R = (1, 1, 1, 2)

	State	Action
1	Good as new	1 Keep
2	Minor deterioration	1 Keep
3	Major deterioration	1 Keep
4	Failed	2 Replace

Discount factor = 0.985222 (rate of return = 1.5%)

Minimizing:

Present value of all future costs i.e., MDP with discounting is used.

Assume a rate of return of 1.5% per month (18% per year)

$$\beta = \frac{1}{1+r} = \frac{1}{1.015} = 0.985222$$

That is, the present value of a \$1 cost next month is \$0.985222



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#### **Value Determination**

Solve the system of equations:

$$\mathbf{v}_i(R) = C_i^{k_i} + \beta \sum_{i \in S} \, \mathbf{p}_{ij}^{k_i} \, \mathbf{v}_j(R) \quad \ \forall \ i \in S$$

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$$P^R = \left[ \begin{array}{cccc} 0.75 & 0.1875 & 0.0625 & 0 \\ 0 & 0.75 & 0.1875 & 0.0625 \\ 0 & 0 & 0.75 & 0.25 \\ 0.75 & 0.1875 & 0.0625 & 0 \end{array} \right]$$

$$\mathbf{v}_i(R) = \mathbf{C}_i^{k_i} + \ \beta \sum_{j \in S} \ \mathbf{p}_{ij}^{k_i} \ \mathbf{v}_j(R) \quad \forall \ i \in S$$

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$$\mathbf{v}_i(R) = \mathbf{C}_i^{k_i} + \beta \sum_{j \in S} \mathbf{p}_{ij}^{k_i} \mathbf{v}_j(R) \quad \forall i \in S$$

 $v_1 = 100 + 0.98522 (0.75 v_1 + 0.1875 v_2 + 0.0625 v_3)$  $v_2 = 200 + 0.98522 (0.75v_2 + 0.1875v_3 + 0.0625v_4)$ 

 $v_3 = 500 + 0.98522 (0.75v_3 + 0.25v_4)$ 

 $v_4 = 3100 + 0.98522 (0.75v_1 + 0.1875v_2 + 0.0625v_3)$ 

Solution:

i	Vi
1 2 3 4	35567.4 36960.8 38299.4 38567.4

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i	٧i
1234	35567.4 36960.8 38299.4 38567.4

That is, \$35,567.40 invested at 1.5% per month interest would sufficient to pay all future operation and replacement cost for the machine, if it is initially in state 1, i.e., "good as new"

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## Policy Improvement

Policy Improvement Step: Evaluation of alternate actions

State #2, Minor deterioration

Current Policy: action #1, Keep

Evaluate alternative action: #2, Replace

$$\mathbf{v}_i' = \mathbf{C}_i^{k'_i} + \beta \sum_i p_{ij}^{k'_i} \mathbf{v}_j$$
 i=2,  $k_i'$ =2

# i V<sub>1</sub> 1 35567.4 2 36960.8 3 38299.4 4 38567.4

$$\mathbf{v}_i' = \mathbf{C}_i^{k'_i} + \beta \sum_j p_{ij}^{k'_i} \mathbf{v}_j$$
 i=2,  $k_i'$ =2

$$\begin{aligned} \mathbf{v}_2' &= 2100 + 0.98522 \ (0.75\mathbf{v}_1 + 0.1875\mathbf{v}_2 + 0.675\mathbf{v}_3) \\ &= 2100 + 0.98522 \ \big(0.75 \!\!\times\!\! 35567.4 + 0.1875 \!\!\times\!\! 36960.8 \\ &\quad + 0.0625 \!\!\times\!\! 38299.4 \big) \end{aligned}$$

= 37567.40

That is, if we are initially in state 2 and replace the machine, but thereafter follow the original policy R, the present value of all future costs is \$27,567.40

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## **Policy Improvement**

State #3, Major deterioration

Current Policy: action #1, Keep

Evaluate the alternate action: Replace

Since  $v_3 < v_3$ , i.e.,  $\Delta v_3 = v_3 - v_3 < 0$ , the policy in this state should be changed to "replace"

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# Policy Improvement

State #2, Minor deterioration

Current Policy: action #1, Keep

k	name	Λ.	ΔV
1	Keep Replace	35128.6 35799.4	0 670.718

 $V^+(k)$  = total discounted cost if action k is selected for one stage, & current policy is followed thereafter

 $\Delta V(k)$  = improvement (if <0)

The policy for this state should not be changed.

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# Policy Improvement

No improvement is possible, so the current policy:

	State	Action	
1	Good as new	1	Keep
2	Minor deterioration	1	Keep
3	Major deterioration	2	Replace
4	Failed	2	Replace

is optimal!

## Policy Improvement

k	name	γ,	ΔV
1 2	Keep Replace	36960.8 37567.4	0 606.62

 $\label{eq:V'(k)} \mbox{$\,$^{\prime\prime}(k)$} = \mbox{total discounted cost if action $k$ is selected for one stage, $\&$ current policy is followed thereafter}$ 

 $\Delta V(k)$  = improvement (if <0)

Since  $v_2 > v_2$ , the current policy for this state should not be changed.

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## **Value Determination**

Discount factor = 0.985222 (rate of return = 1.5%)

New policy:

State	Action
1 Good as new 2 Minor deterioration 3 Major deterioration 4 Failed	1 Keep 1 Keep 2 Replace 2 Replace

New present values:

i	٧i	
1 2 3 4	33799.4 35128.6 36299.4 36799.4	000000000000000000000000000000000000000

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# Policy Improvement

State #3, Major deterioration

Current Policy: action #2, Replace

k	name	Λ.	ΔV
1	Keep Replace	36386.1 36299.4	86.709 0

 $\label{eq:V'(k)} \mbox{$\,$} \mb$ 

 $\Delta V(k)$  = improvement (if <0)

The policy for this state should not be changed.

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