Electric Generator Control MDP Example

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An electricity plant has two generators for generating electricity.
The demand fluctuates during the day, which is divided into 4-hour blocks of time.
d_k = demand during block k, k=1,2,3,4,5,6

An excess of electricity produced cannot be stored.
c_j = capacity (kWh /4hr) of electricity of generator j
r_j = operating cost/4hr of generator j
S_j = setup cost to turn on an idle generator

What control rule will minimize the daily cost?

Data

k	1	2	3	4	5	6
d_k	20	40	60	90	70	30

j	c_j	S_j	r_j
1	40	500	1000
2	60	300	1100

States (t,m) where
t = period of the day   t=1,2,...6;
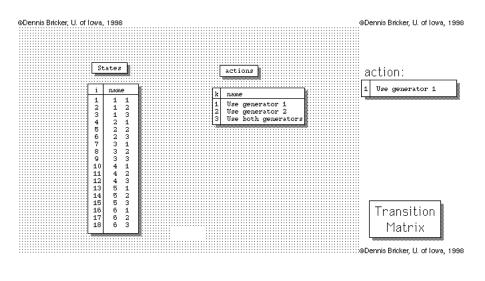m indicates which generators are on:
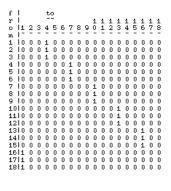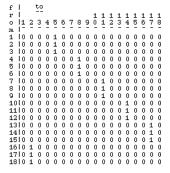1: generator 1 only
2: generator 2 only
3: both generators

Actions   k =
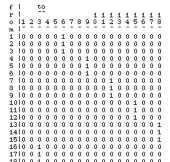1: generator 1 only
2: generator 2 only
3: both generators

States

i	name	
1	1	1
2	1	2
3	1	3
4	2	1
5	2	2
6	2	3
7	3	1
8	3	2
9	3	3
10	4	1
11	4	2
12	4	3
13	5	1
14	5	2
15	5	3
16	6	1
17	6	2
18	6	3

actions

k	name
1	Use generator 1
2	Use generator 2
3	Use both generators

action: 1 Use generator 1

Transition Matrix

```
f |      to
r |                    1 1 1 1 1 1 1 1 1
o |1 2 3 4 5 6 7 8 9 0 1 2 3 4 5 6 7 8
m |-------------------------------------
 1 |0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0
 2 |0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0
 3 |0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0
 4 |0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0
 5 |0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0
 6 |0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0
 7 |0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0
 8 |0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0
 9 |0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0
10 |0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0
11 |0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0
12 |0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0
13 |0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0
14 |0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0
15 |0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0
16 |1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
17 |1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
18 |1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
```

action: 2 Use generator 2

Transition Matrix

```
f |      to
r |                    1 1 1 1 1 1 1 1 1
o |1 2 3 4 5 6 7 8 9 0 1 2 3 4 5 6 7 8
m |-------------------------------------
 1 |0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0
 2 |0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0
 3 |0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0
 4 |0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0
 5 |0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0
 6 |0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0
 7 |0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0
 8 |0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0
 9 |0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0
10 |0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0
11 |0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0
12 |0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0
13 |0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0
14 |0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0
15 |0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0
16 |0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
17 |0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
18 |0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
```

action: 3 Use both generators

Transition Matrix

```
f |      to
r |                    1 1 1 1 1 1 1 1 1
o |1 2 3 4 5 6 7 8 9 0 1 2 3 4 5 6 7 8
m |-------------------------------------
 1 |0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0
 2 |0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0
 3 |0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0
 4 |0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0
 5 |0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0
 6 |0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0
 7 |0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0
 8 |0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0
 9 |0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0
10 |0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0
11 |0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0
12 |0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0
13 |0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1
14 |0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1
15 |0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1
16 |0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
17 |0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
18 |0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
```

```
k:  |  1    2    3    1    2    3    1    2    3    1    2    3    1    2
    |  ---  ---  ---  ---  ---  ---  ---  ---  ---  ---  ---  ---  ---  ---
i:  |  1    1    1    2    2    2    3    3    3    4    4    4    5    5
    |  ---  ---  ---  ---  ---  ---  ---  ---  ---  ---  ---  ---  ---  ---
Min | 1000 1400 2400 1500 1100 2600 1000 1100 2100 1000 1400 2400 1500 1100
    |  1    1    1    0    0    0    0    0    0    0    0    0    0    0
    |  0    0    0    1    1    1    0    0    0    0    0    0    0    0
    |  0    0    0    0    0    0    1    1    1    0    0    0    0    0
    | -1    0    0   -1    0    0   -1    0    0    1    1    1    0    0
    |  0   -1    0    0   -1    0    0   -1    0    0    0    0    1    1
    |  0    0   -1    0    0   -1    0    0   -1    0    0    0    0    0
    |  0    0    0    0    0    0    0    0    0   -1    0    0   -1    0
    |  0    0    0    0    0    0    0    0    0    0   -1    0    0   -1
    |  0    0    0    0    0    0    0    0    0    0    0   -1    0    0
    |  0    0    0    0    0    0    0    0    0    0    0    0    0    0
    |  0    0    0    0    0    0    0    0    0    0    0    0    0    0
    |  0    0    0    0    0    0    0    0    0    0    0    0    0    0
    |  0    0    0    0    0    0    0    0    0    0    0    0    0    0
    |  0    0    0    0    0    0    0    0    0    0    0    0    0    0
    |  0    0    0    0    0    0    0    0    0    0    0    0    0    0
    |  0    0    0    0    0    0    0    0    0    0    0    0    0    0
    |  1    1    1    1    1    1    1    1    1    1    1    1    1    1
```

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```
   3    1    2    3    2    3    2    3    2    3    3    3    3    3    3    3
  ---  ---  ---  ---  ---  ---  ---  ---  ---  ---  ---  ---  ---  ---  ---  ---
   5    6    6    6    7    7    8    8    9    9   10   11   12   13   14   15
  ---  ---  ---  ---  ---  ---  ---  ---  ---  ---  ---  ---  ---  ---  ---  ---
  2600 1000 1100 2100 1400 2400 1100 2600 1100 2100 2400 2600 2100 2400 2600 2100
   0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0
   0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0
   0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0
   1    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0
   0    1    1    1    0    0    0    0    0    0    0    0    0    0    0    0
   0   -1    0    0    1    1    0    0    0    0    0    0    0    0    0    0
   0    0   -1    0    0    0    1    1    0    0    0    0    0    0    0    0
  -1    0    0   -1    0    0    1    1    0    0    0    0    0    0    0    0
   0    0    0    0    0    0    0    0    1    1    0    0    0    0    0    0
   0    0    0    0   -1    0   -1    0   -1    0    0    1    0    0    0    0
   0    0    0    0    0   -1    0   -1    0   -1    0    0    1    0    0    0
   0    0    0    0    0    0    0    0    0    0    0    0    0    1    0    0
   0    0    0    0    0    0    0    0    0    0   -1   -1   -1    0    0    1
   0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0
   0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0
   1    1    1    1    1    1    1    1    1    1    1    1    1    1    1    1
```

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```
   1    2    3    1    2    3    1    2    3  | R
  ---  ---  ---  ---  ---  ---  ---  ---  --- | H
  16   16   16   17   17   17   18   18   18  | S
  ---  ---  ---  ---  ---  ---  ---  ---  --- | -
  1000 1400 2400 1500 1100 2600 1000 1100 2100 |
  -1    0    0   -1    0    0   -1    0    0  | 0
   0   -1    0    0   -1    0    0   -1    0  | 0
   0    0   -1    0    0   -1    0    0   -1  | 0
   0    0    0    0    0    0    0    0    0  | 0
   0    0    0    0    0    0    0    0    0  | 0
   0    0    0    0    0    0    0    0    0  | 0
   0    0    0    0    0    0    0    0    0  | 0
   0    0    0    0    0    0    0    0    0  | 0
   0    0    0    0    0    0    0    0    0  | 0
   0    0    0    0    0    0    0    0    0  | 0
   0    0    0    0    0    0    0    0    0  | 0
   0    0    0    0    0    0    0    0    0  | 0
   0    0    0    0    0    0    0    0    0  | 0
   1    1    1    0    0    0    0    0    0  | 0
   0    0    0    1    1    1    0    0    0  | 0
   1    1    1    1    1    1    1    1    1  | 1
```

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```
            State |         action
            ----- | ---------------------
              1  1 | Use both generators
              1  2 | Use both generators
              1  3 | Use both generators
              2  1 | Use both generators
              2  2 | Use both generators
              2  3 | Use both generators
              3  1 | Use both generators
              3  2 | Use both generators
              3  3 | Use both generators
              4  1 | Use both generators
              4  2 | Use both generators
              4  3 | Use both generators
              5  1 | Use both generators
              5  2 | Use both generators
              5  3 | Use both generators
              6  1 | Use both generators
              6  2 | Use both generators
              6  3 | Use both generators
```

Policy

The average cost/period in steady state is 2100

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```
# of states: 18
# of actions: 3
                            Cost Matrix
                            -----------

                         1    2    3    4    5    6     7       8       9
                        ---  ---  ---  ---  ---  ---  ------- ------- -------
|k| name               |
|1| Use generator 1     | 1000 1500 1000 1000 1500 1000 1001000 1001500 1001000
|2| Use generator 2     | 1400 1100 1100 1400 1100 1100    1400    1100    1100
|3| Use both generators | 2400 2600 2100 2400 2600 2100    2400    2600    2100

            10       11       12       13       14       15      16   17   18
          ------- ------- ------- ------- ------- ------- ------- ---- ----
          1001000 1001500 1001000 1001000 1001500 1001000 1000 1500 1000
          1001400 1001100 1001100 1001400 1001100 1001100 1400 1100 1100
             2400    2600    2100    2400    2600    2100 2400 2600 2100

(Rows ~ actions, Columns ~ states)


A value of 999999 above signals
an infeasible action in a state.
```

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