Batch Processing Problem
Markov Decision Model

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Each minute, either zero, one, or two customers arrive at a facility which has a capacity (batch size) of 5.

i=#waiting	0	1	2	3	4
P{0 arrivals}	0.04	0.25	0.44	0.61	0.76
P{1 arrival}	0.6	0.5	0.4	0.3	0.2
P{2 arrivals}	0.36	0.25	0.16	0.09	0.04

(Probability of arrival diminishes as the queue lengthens... probability of 2 arrivals is square of probability of one.)

There is a "holding cost" per customer of 10¢ per minute, and a processing cost of 40¢ per batch *(independent of batch size).*

What is the optimal queue length for batch processing?

(The smaller the batches, the more the processing cost, while the larger the batches, the more the holding cost.)

Markov Decision Model

States

i	name
1	0 waiting
2	1 waiting
3	2 waiting
4	3 waiting
5	4 waiting
6	≥5 waiting

Actions

k	name
1	wait another minute
2	process the batch

state defined as number in the queue at the beginning of the stage (minute) before any arrivals

Cost Matrix

name	1	2	3	4	5	6
wait another minute	0	0.1	0.2	0.3	0.4	999
process batch	0.4	0.5	0.6	0.7	0.8	0.9

includes holding cost for customers in queue at beginning of stage

A value of 999 above signals
an infeasible action in a state.

(Rows ~ actions, Columns ~ states)

Transition Probabilities

Action: wait another minute

from \ to	1	2	3	4	5	6
1	0.04	0.6	0.36	0	0	0
2	0	0.25	0.5	0.25	0	0
3	0	0	0.44	0.4	0.16	0
4	0	0	0	0.61	0.3	0.09
5	0	0	0	0	0.76	0.24
	0	0	0	0	0	1

Action: process the batch

from \ to	1	2	3	4	5	6
1	0.04	0.6	0.36	0	0	0
2	0.04	0.6	0.36	0	0	0
3	0.04	0.6	0.36	0	0	0
4	0.04	0.6	0.36	0	0	0
5	0.04	0.6	0.36	0	0	0
	0.04	0.6	0.36	0	0	0

LP Tableau

k	1	2	1	2	1	2	1	2	1	2	2	RHS
i	1	1	2	2	3	3	4	4	5	5	6	
	0	0.4	0.1	0.5	0.2	0.6	0.3	0.7	0.4	0.8	0.9	
	0.96	0.96	0	¯0.04	0	¯0.04	0	¯0.04	0	¯0.04	¯0.04	0
	¯0.6	¯0.6	0.75	0.4	0	¯0.6	0	¯0.6	0	¯0.6	¯0.6	0
	¯0.36	¯0.36	¯0.5	¯0.36	0.56	0.64	0	¯0.36	0	¯0.36	¯0.36	0
	0	0	¯0.25	0	¯0.4	0	0.39	1	0	0	0	0
	0	0	0	0	¯0.16	0	¯0.3	0	0.24	1	0	0
	1	1	1	1	1	1	1	1	1	1	1	1

*initial basic solution corresponds to the policy:
wait until 5 customers arrive
before processing a batch*

	☆		☆		☆		☆		☆		☆	
k:	1	2	1	2	1	2	1	2	1	2	2	
i:	1	1	2	2	3	3	4	4	5	5	6	rhs
Min	0	0.4	0	0.2180	0	0.0380	0	¯0.1080	0	¯0.2187	0	¯0.3715
	1	1	0	¯0.0382	0	¯0.0340	0	¯0.0276	0	¯0.0199	0	0.0047
	0	0	1	0.5678	0	¯0.6801	0	¯0.5528	0	¯0.3993	0	0.0958
	0	0	0	¯0.1081	1	0.6318	0	¯0.9379	0	¯0.6774	0	0.1625
	0	0	0	0.2530	0	0.2120	1	1.2476	0	¯0.9507	0	0.2281
	0	0	0	0.2441	0	0.6863	0	0.9342	1	2.5266	0	0.3936
	0	0	0	0.0813	0	0.1838	0	0.3365	0	0.5208	1	0.1150

Iteration 0

	☆		☆		☆		☆		☆		☆	
k:	1	2	1	2	1	2	1	2	1	2	2	
i:	1	1	2	2	3	3	4	4	5	5	6	rhs
Min	0	0.4	0	0.2180	0	0.0380	0	-0.1080	0	-0.2187	0	-0.3715
	1	1	0	-0.0382	0	-0.0340	0	-0.0276	0	-0.0199	0	0.0047
	0	0	1	0.5678	0	-0.6801	0	-0.5528	0	-0.3993	0	0.0958
	0	0	0	-0.1081	1	0.6318	0	-0.9379	0	-0.6774	0	0.1625
	0	0	0	0.2530	0	0.2120	1	1.2476	0	-0.9507	0	0.2281
	0	0	0	0.2441	0	0.6863	0	0.9342	1	(2.5266)	0	0.3936
	0	0	0	0.0813	0	0.1838	0	0.3365	0	0.5208	1	0.1150

Iteration 0

↑ pivot column

initial basic solution corresponds to the policy: wait until 5 customers arrive before processing a batch

Iteration 0

Policy: (Cost= 0.3715)

steadystate distribution ↓

	State	Action	P{i}
1	0 waiting	1 wait another minute	0.00479179
2	1 waiting	1 wait another minute	0.0958357
3	2 waiting	1 wait another minute	0.162578
4	3 waiting	1 wait another minute	0.22818
5	4 waiting	1 wait another minute	0.393611
6	≥5 waiting	2 process batch	0.115003

Iteration 1

	☆		☆		☆		☆			☆	☆	
k:	1	2	1	2	1	2	1	2	1	2	2	
i:	1	1	2	2	3	3	4	4	5	5	6	rhs
Min	0	0.4	0	0.2391	0	0.0974	0	-0.0271	0.0865	0	0	-0.3374
	1	1	0	-0.0363	0	-0.0285	0	-0.0202	0.0079	0	0	0.0079
	0	0	1	0.6064	0	-0.5716	0	-0.4052	0.1580	0	0	0.1580
	0	0	0	-0.0426	1	0.8159	0	-0.6874	0.2681	0	0	0.2681
	0	0	0	0.3449	0	0.4703	1	1.5992	0.3762	0	0	0.3762
	0	0	0	0.0966	0	0.2716	0	0.3697	0.3957	1	0	0.1557
	0	0	0	0.0310	0	0.0423	0	0.1439	-0.2061	0	1	0.0338

Iteration 1

	☆		☆		☆		☆			☆	☆	
k:	1	2	1	2	1	2	1	2	1	2	2	
i:	1	1	2	2	3	3	4	4	5	5	6	rhs
Min	0	0.4	0	0.2391	0	0.0974	0	-0.0271	0.0865	0	0	-0.3374
	1	1	0	-0.0363	0	-0.0285	0	-0.0202	0.0079	0	0	0.0079
	0	0	1	0.6064	0	-0.5716	0	-0.4052	0.1580	0	0	0.1580
	0	0	0	-0.0426	1	0.8159	0	-0.6874	0.2681	0	0	0.2681
	0	0	0	0.3449	0	0.4703	1	(1.5992)	0.3762	0	0	0.3762
	0	0	0	0.0966	0	0.2716	0	0.3697	0.3957	1	0	0.1557
	0	0	0	0.0310	0	0.0423	0	0.1439	-0.2061	0	1	0.0338

↑ pivot column

Iteration 1

Policy: (Cost= 0.337422)

	State	Action	P{i}
1	0 waiting	1 wait another minute	0.00790216
2	1 waiting	1 wait another minute	0.158043
3	2 waiting	1 wait another minute	0.268109
4	3 waiting	1 wait another minute	0.376294
5	4 waiting	2 process batch	0.155786
6	≥5 waiting	2 process batch	0.0338664

Iteration 2

	☆		☆		☆			☆		☆	☆	
k:	1	2	1	2	1	2	1	2	1	2	2	
i:	1	1	2	2	3	3	4	4	5	5	6	rhs
Min	0	0.4	0	0.2450	0	0.1054	0.0169	0	0.0929	0	0	-0.33104
	1	1	0	-0.0319	0	-0.0226	0.0126	0	0.0126	0	0	0.01266
	0	0	1	0.6938	0	-0.4524	0.2533	0	0.2533	0	0	0.25339
	0	0	0	0.1055	1	1.0181	0.4298	0	0.4298	0	0	0.42986
	0	0	0	0.2156	0	0.2941	0.6252	1	0.2352	0	0	0.23529
	0	0	0	0.0168	0	0.1628	-0.2312	0	0.3087	1	0	0.06877
	0	0	0	0	0	0	-0.09	0	-0.24	0	1	0

Reduced costs are nonnegative!

Optimal Policy!

Iteration 2

Policy: (Cost= 0.331041)

	State	Action	P{i}
1	0 waiting	1 wait another minute	0.0126697
2	1 waiting	1 wait another minute	0.253394
3	2 waiting	1 wait another minute	0.429864
4	3 waiting	2 process batch	0.235294
5	4 waiting	2 process batch	0.0687783
6	≥5 waiting	2 process batch	0

Optimal Policy!