

Each minute, either zero, one, or two customers arrive at a facility which has a capacity (batch size) of 5.

| i=#waiting    | 0    | 1    | 2    | 3    | 4    |
|---------------|------|------|------|------|------|
| P{0 arrivals} | 0.04 | 0.25 | 0.44 | 0.61 | 0.76 |
| P{1 arrival}  | 0.6  | 0.5  | 0.4  | 0.3  | 0.2  |
| P{2 arrivals} | 0.36 | 0.25 | 0.16 | 0.09 | 0.04 |

(Probability of arrival diminishes as the queue lengthens... probability of 2 arrivals is square of probability of one.)

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There is a "holding cost" per customer of 10¢ per minute, and a processing cost of 40¢ per batch (independent of batch size).

# What is the optimal queue length for batch processing?

(The smaller the batches, the more the processing cost, while the larger the batches, the more the holding cost.)

# Markov Decision Model

States

i name
1 0 waiting
2 1 waiting
3 2 waiting
4 3 waiting
5 4 waiting
6 ≥5 waiting

Actions

k name
1 wait another minute
2 process the batch

state defined as number in the queue at the beginning of the stage (minute) before any arrivals

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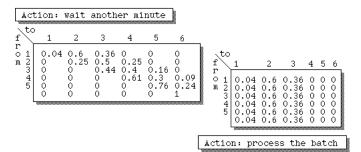
## Cost Matrix

| 1 0 2 0 2  | 000         |
|------------|-------------|
| .1 0.2 0.3 | 0.4 999     |
| .5 0.6 0.7 | 0.8 0.9     |
| ,          | 0.5 0.6 0.7 |

A value of 999 above signals an infeasible action in a state.

(Rows ~ actions, Columns ~ states)

includes holding cost for customers in queue at beginning of stage Transition Probabilities



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# LP Tableau

| k 1   | 2     | 1     | 2     | 1     | 2     | 1    | 2     | 1    | 2     | 2     | R      |
|-------|-------|-------|-------|-------|-------|------|-------|------|-------|-------|--------|
| i 1   | 1     | 2     | 2     | 3     | 3     | 4    | 4     | 5    | 5     | 6     | H<br>S |
| 0     | 0.4   | 0.1   | 0.5   | 0.2   | 0.6   | 0.3  | 0.7   | 0.4  | 0.8   | 0.9   |        |
| 0.96  | 0.96  | 0     | -0.04 | 0     | -0.04 | 0    | -0.04 | 0    | -0.04 | -0.04 | 0      |
|       | -0.6  |       | 0.4   |       |       | 0    | -0.6  | 0    | -0.6  | -0.6  | 0      |
| -0.36 | -0.36 |       |       |       |       |      | -0.36 | Ō.   | -0.36 | -0.36 | 0      |
| ļĢ    | 0     | -0.25 | 0     | -0.4  | 0     | 0.39 |       | 0    | 0     | 0     | 0      |
| 0     | 0     | 0     | 0     | -0.16 | 0     | -0.3 | 0     | 0.24 | 1     | 0     | 0      |
| 1     | 1     | 1     | 1     | 1     | 1     | 1    | 1     | 1    | 1     | 1     | 1      |

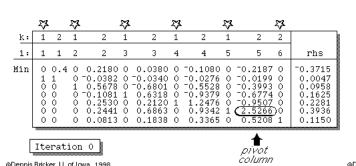
initial basic solution corresponds to the policy: wait until 5 customers arrive before processing a batch

|     | Ŋ                          |                              | ¥ |        | Ŋ |  | Ŋ |  | ¥    |   | ×                          |   |
|-----|----------------------------|------------------------------|---|--------|---|--|---|--|------|---|----------------------------|---|
| k:  | 1                          | 2                            | 1 | 2      | 1 | 2  | 1 | 2                                      | 1    | 2                                       | 2                          |   |
| i:  | 1                          | 1                            | 2 | 2      | 3 | 3  | 4 | 4                                      | 5    | 5                                       | 6                          | rhs   |
| Min | 0<br>1<br>0<br>0<br>0<br>0 | 0.4<br>1<br>0<br>0<br>0<br>0 | - | 0.2441 | - | -0.0340<br>-0.6801<br>0.6318<br>0.2120<br>0.6863 | 0 | -0.5528<br>-0.9379<br>1.2476<br>0.9342 | 0000 | -0.3993<br>-0.6774<br>-0.9507<br>2.5266 | 0<br>0<br>0<br>0<br>0<br>0 | -0.3715<br>0.0047<br>0.0958<br>0.1625<br>0.2281<br>0.3936<br>0.1150 |

Iteration 0

# initial basic solution corresponds to the policy: wait until 5 customers arrive before processing a batch

-steadystate



distribution Policy: (Cost= 0.3715 ) Action P{i} State 0 waiting 1 waiting 2 waiting 3 waiting 4 waiting ≥5 waiting wait another minute process batch 0.00479179 0.0958357 0.162578 0.22818 0.393611 0.115003

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Iteration 0

|  | Iteration | 1 |
|--|-----------|---|
|--|-----------|---|

|         | Ŋ           |                              | Ŋ                          |  | ¥                          |  | 23            |  |  | Ŋ                               | ×                          |   |
|---------|-------------|------------------------------|----------------------------|--|----------------------------|--|---------------|--|--|---------------------------------|----------------------------|---|
| k:      | 1           | 2                            | 1                          | 2  | 1                          | 2  | 1             | 2  | 1  | 2                               | 2                          |   |
| i:      | 1           | 1                            | 2                          | 2  | 3                          | 3  | 4             | 4  | 5  | 5                               | 6                          | rhs   |
| <br>Min | 0 1 0 0 0 0 | 0.4<br>1<br>0<br>0<br>0<br>0 | 0<br>0<br>1<br>0<br>0<br>0 | 0.2391<br>-0.0363<br>0.6064<br>-0.0426<br>0.3449<br>0.0966<br>0.0310 | 0<br>0<br>0<br>1<br>0<br>0 | 0.0974<br>-0.0285<br>-0.5716<br>0.8159<br>0.4703<br>0.2716<br>0.0423 | 0 0 0 0 1 0 0 | -0.0271<br>-0.0202<br>-0.4052<br>-0.6874<br>1.5992<br>0.3697<br>0.1439 | 0.0865<br>0.0079<br>0.1580<br>0.2681<br>0.3762<br>0.3957 | 0<br>0<br>0<br>0<br>0<br>1<br>0 | 0<br>0<br>0<br>0<br>0<br>0 | -0.3374<br>0.0079<br>0.1580<br>0.2681<br>0.3762<br>0.1557<br>0.0338 |

### Iteration 1

|     | 23 |     | ¥ |                   | Ŋ |                    | Ŋ |          |        | Ŋ | ₩. |                  |
|-----|----|-----|---|-------------------|---|--------------------|---|----------|--------|---|----|------------------|
| k:  | 1  | 2   | 1 | 2                 | 1 | 2                  | 1 | 2        | 1      | 2 | 2  |                  |
| i:  | 1  | 1   | 2 | 2                 | 3 | 3                  | 4 | 4        | 5      | 5 | 6  | rhs              |
| Min | 0  | 0.4 | - | 0.2391            | 0 | 0.0974             | - | -0.0271  | 0.0865 | 0 | 0  | -0.3374          |
|     | 0  | 0   | 1 | -0.0363<br>0.6064 | 0 | -0.0285<br>-0.5716 | 0 | -0.4052  | 0.0079 | 0 | 0  | 0.0079<br>0.1580 |
|     | 0  | 0   | 0 | -0.0426<br>0.3449 | 0 | 0.8159<br>0.4703   | 1 | (1.5992) | 0.2681 | 0 | 0  | 0.2681<br>0.3762 |
|     | 0  | 0   | 0 | 0.0966            | 0 | 0.2716             | 0 | 0.3697   | 0.3957 | 1 | 0  | 0.1557<br>0.0338 |
|     |    | -   | - |                   |   |                    |   | <b>A</b> |        |   | _  |                  |

ੈ pivot column

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### Iteration 1

Policy: (Cost= 0.337422 )

|   | State  | Action                | P{i}       |
|---|--|-----------------------|------------|
| 1 | 0 waiting 1 waiting 2 waiting 3 waiting 4 waiting ≥5 waiting | 1 wait another minute | 0.00790216 |
| 2 |  | 1 wait another minute | 0.158043   |
| 3 |  | 1 wait another minute | 0.268109   |
| 4 |  | 1 wait another minute | 0.376294   |
| 5 |  | 2 process batch       | 0.155786   |
| 6 |  | 2 process batch       | 0.0338664  |

### Iteration 2

|     | Ŋ                          |                              | Ŋ                          |  | Ŋ                          |   |   | Ŋ                          |  | Ŋ                          | X.                         |  |
|-----|----------------------------|------------------------------|----------------------------|--|----------------------------|---|---|----------------------------|--|----------------------------|----------------------------|--|
| k:  | 1                          | 2                            | 1                          | 2  | 1                          | 2   | 1   | 2                          | 1  | 2                          | 2                          |  |
| i:  | 1                          | 1                            | 2                          | 2  | 3                          | 3   | 4   | 4                          | 5  | 5                          | 6                          | rhs  |
| Min | 0<br>1<br>0<br>0<br>0<br>0 | 0.4<br>1<br>0<br>0<br>0<br>0 | 0<br>0<br>1<br>0<br>0<br>0 | 0.2450<br>-0.0319<br>0.6938<br>0.1055<br>0.2156<br>0.0168<br>0 | 0<br>0<br>0<br>1<br>0<br>0 | 0.1054<br>-0.0226<br>-0.4524<br>1.0181<br>0.2941<br>0.1628<br>0 | 0.0169<br>0.0126<br>0.2533<br>0.4298<br>0.6252<br>-0.2312 | 0<br>0<br>0<br>0<br>1<br>0 | 0.0929<br>0.0126<br>0.2533<br>0.4298<br>0.2352<br>0.3087 | 0<br>0<br>0<br>0<br>0<br>1 | 0<br>0<br>0<br>0<br>0<br>0 | -0.33104<br>0.01266<br>0.25339<br>0.42986<br>0.23529<br>0.06877<br>0 |

Reduced costs are nonnegative!

Optimal Policy!

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Iteration 2

Policy: (Cost= 0.331041 )

|                            | State  | Action   | P{i}   |
|----------------------------|--|--|--|
| 1<br>2<br>3<br>4<br>5<br>6 | 0 waiting 1 waiting 2 waiting 3 waiting 4 waiting ≥5 waiting | 1 wait another minute<br>1 wait another minute<br>1 wait another minute<br>2 process batch<br>2 process batch<br>2 process batch | 0.0126697<br>0.253394<br>0.429864<br>0.235294<br>0.0687783 |

### Optimal Policy!

