Sensitivity Analysis in Geometric Programming

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Approximating the Effects of Inflation

Suppose that we have found the optimal solution of a GP problem, and we wish to test its sensitivity to an increase in a cost coefficient C .

Let

known
- C_t = original cost coefficient
- δ^* = optimal dual solution of original problem
- y^* = optimal cost of original problem

unknown
- C'_t = cost coefficient after inflation
- $y^{*\prime}$ = optimal cost after inflation

$$y^* = \prod_{i=1}^{N}\left(\frac{C_i}{\delta_i^*}\right)^{\delta_i^*}\prod_{k=0}^{p}\lambda_k^{*\lambda_k^*}$$

$$=\left(\frac{C_t}{\delta_t^*}\cdot\frac{C'_t}{C'_t}\right)^{\delta_t^*}\prod_{\substack{i=1\\ i\neq t}}^{N}\left(\frac{C_i}{\delta_i^*}\right)^{\delta_i^*}\prod_{k=0}^{p}\lambda_k^{*\lambda_k^*}=\left(\frac{C_t}{C'_t}\cdot\frac{C'_t}{\delta_t^*}\right)^{\delta_t^*}\prod_{\substack{i=1\\ i\neq t}}^{N}\left(\frac{C_i}{\delta_i^*}\right)^{\delta_i^*}\prod_{k=0}^{p}\lambda_k^{*\lambda_k^*}$$

$$=\left(\frac{C_t}{C'_t}\right)^{\delta_t^*}\underbrace{\left(\frac{C'_t}{\delta_t^*}\right)^{\delta_t^*}\prod_{\substack{i=1\\ i\neq t}}^{N}\left(\frac{C_i}{\delta_i^*}\right)^{\delta_i^*}\prod_{k=0}^{p}\lambda_k^{*\lambda_k^*}}\leq\left(\frac{C_t}{C'_t}\right)^{\delta_t^*}y^{*\prime}$$

Dual objective value of new problem, evaluated at the original dual solution

$$y^* \leq \left(\frac{C_t}{C'_t}\right)^{\delta_t^*} y^{*\prime} \Rightarrow y^{*\prime} \geq \left(\frac{C'_t}{C_t}\right)^{\delta_t^*} y^*$$

That is, the minimum cost of the problem has increased by *at least* a factor of $\left(\frac{C'_t}{C_t}\right)^{\delta_t^*}$

Example

Cargo Ship Design Problem

The optimal cost is $71517156

The optimal dual variables for terms in the objective function are

$\delta_1^* = 0.129647$	*cost of power plants*
$\delta_2^* = 0.700782$	*cost of ships*
$\delta_3^* = 0.16957$	*cost of fuel*

Estimate the optimal cost if the cost of fuel were to increase by 20%.

The optimal primal variables are

14.9566	*number of ships*
16754.6	*cargo load capacity (tons)*
12.4553	*speed (knots)*
0.877868	*fraction of time en route*

Lower Bound

$$y^{*\prime} \geq \left(\frac{C'_t}{C_t}\right)^{\delta_t^*} y^*$$

$$y^{*\prime} \geq (1.2)^{0.16957} \times 71517156$$
$$= 1.0314 \times 71517156 = 73762736$$

i.e., the cost will increase by at least 3.14%.

Upper Bound

Since the previous solution is still feasible, it provides us with an upper bound on the new cost: 73982323

That is, the cost should not increase more than 3.447%

(since $\frac{73982323}{71517156} = 1.03447$)

Thus, we are able to estimate that

$$73762736 \leq y^{*\prime} \leq 73982323$$

If we solve the GP with the new cost coefficient for fuel, we obtain the minimum cost 73897491 an increase of 3.328%.

The optimal primal variables are now

14.9556	*number of ships*
17383.5	*cargo load capacity (tons)*
12.0047	*speed (knots)*
0.877928	*fraction of time en route*