Dynamic Programming

Process Plan Selection

Considering Sequence-Dependent Setup Costs

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Manufacture of a product requires *four* operations,
each of which may be performed
on any of *three* alternative machines.

The operation cost/unit for the various machines are:

Machine	Operation 1	Operation 2	Operation 3	Operation 4
A	3	4	3	6
B	2	4	5	5
C	4	1	6	4

There is a cost associated with *moving* the product from one machine to another between operations.

These sequence-dependent setup costs are:

From	To	Setup Cost
A	B	2
A	C	1
B	A	2
B	C	1
C	A	2
C	B	1

The total cost of the sequence A→B→B→C is, for example,
3×L + (2 + 4×L) + (5×L) + (1 + 4×L)

DYNAMIC PROGRAMMING MODEL

Let $C^c_{s,x}$ = cost of *changing* part from machine s to machine x

$C^p_{n,x}$ = *processing* cost per unit for operation n on machine x

L = *lot size*

Stages: n = operation $(n=1, 2, \ldots N)$

State: s_n = machine on which previous operation $(n-1)$ was performed

Decision: x_n = machine on which operation n is to be performed

Optimal value function

$f_n(s_n)$ = minimum cost of completing operations $n, n+1, \ldots N$
if the part is currently loaded on machine s_n.

$$f_n(s) = \min\left\{C^c_{s,x} + L \times C^p_{s,x} + f_{n+1}(x)\right\}$$

$$f_N(s) = 0$$

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Setting lot size L = 1, we obtain:

Stage 4---

s \ x:	1	2	3	Min
1	6	7	5	5
2	8	5	5	5
3	8	6	4	4

Stage 3---

s \ x:	1	2	3	Min
1	8	12	11	8
2	10	10	11	10
3	10	11	10	10

Stage 2---

s \ x:	1	2	3	Min
1	12	16	12	12
2	14	14	12	12
3	14	15	11	11

Stage 1---

s \ x:	1	2	3	Min
1	15	16	16	15
2	17	14	16	**14** ←
3	17	15	15	15

The optimal beginning state is #2 (machine B).

Optimal Returns & Decisions

Stage 1

Current State	Optimal Decision	Optimal Value	Next State
A	A	15	A
B	B	14	B
C	B	15	B
	C		C

Stage 2

Current State	Optimal Decision	Optimal Value	Next State
A	A	12	A
	C		C
B	C	12	C
C	C	11	C

Stage 3

Current State	Optimal Decision	Optimal Value	Next State
A	A	8	A
B	A	10	A
	B		B
C	A	10	A
	C		C

Stage 4

Current State	Optimal Decision	Optimal Value	Next State
A	C	5	C
B	B	5	B
	C		C
C	C	4	C

*The minimum cost is achieved by initially loading the parts on machine **B**, resulting in total cost of $14.*
The optimal sequence: **B → C → A → C**

Optimal Solution No. 1

stage	state	decision
1	B	B
2	B	C
3	C	A
4	A	C
5	C	

B → C → A → C

Optimal Solution No. 2

stage	state	decision
1	B	B
2	B	C
3	C	C
4	C	C
5	C	

B → C → C → C
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What is the optimal plan if the lotsize is L=2?

Operation #4:

	A	B	C	min
A				
B				
C				

Operation #2:

	A	B	C	min
A				
B				
C				

Operation #3:

	A	B	C	min
A				
B				
C				

Operation #1:

	A	B	C	min
A				
B				
C				