Machine Replacement Problem via DP

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Suppose that a new car costs $10,000, and that the annual operating cost & resale value are as follows:

Age of car (yrs)	Resale Value	Operating cost in previous year
1	$7000	$300
2	$6000	$500
3	$4000	$800
4	$3000	$1200
5	$2000	$1600
6	$1000	$2200

Starting with a new car, what is the replacement policy that minimizes the net cost of owning and operating a car for the next six years?

(Assuming that
- *initial car has already been paid for*
- *no car is needed at the end of the sixth year)*

Optimal Machine Replacement Problem

Purchase price of new machine = 10000

Age	Maintenance cost prev. yr	Salvage value
1	300	7000
2	500	6000
3	800	4000
4	1200	3000
5	1600	2000
6	2200	1000

G(t) = minimum total cost incurred from time t until the end of the planning horizon, if a new machine has just been purchased.

X*(t) = optimal replacement time for a machine which has been purchased at the beginning of period t.

$$G(t) = \min_{t+1 \le x \le T} \left\{ \sum_{i=1}^{x-t} C_i - S_{x-t} + P_x + G(x) \right\}$$

where

P_t = purchase price of a new machine at time t ($P_T = 0$)

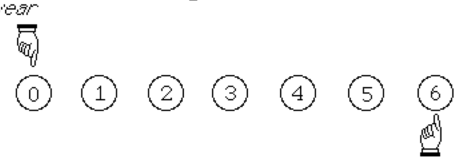
C_i = cost of operation & maintenance of a machine in its i^{th} year

S_j = salvage value of machine of age j

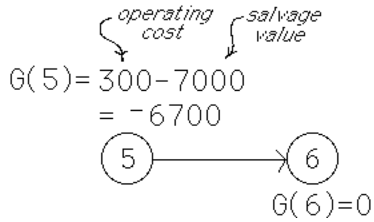
$$G(t) = \min_{t+1 \le x \le T} \left\{ \sum_{i=1}^{x-t} C_i - S_{x-t} + P_x + G(x) \right\}$$

Starting point:
New car at the beginning of the first year

What's the least-cost way to get from node 0 to node 6?



Termination:
No car at the end of the sixth year



Stage 6

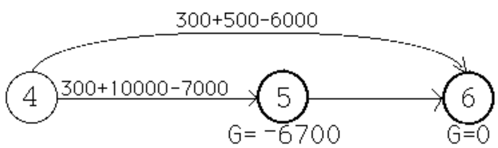
G[6]= 0

Stage 5

Computation of G[5] = Minimum total cost until end of time period 6, given a new machine at time 5

x	C	C+G
6	¯6700	¯6700

X*[5]= 6 = optimal replacement time
G[5]= ¯6700



G(4) = minimum {300 +10000 - 7000 +G(5),
300+500 - 6000 + G(6)}

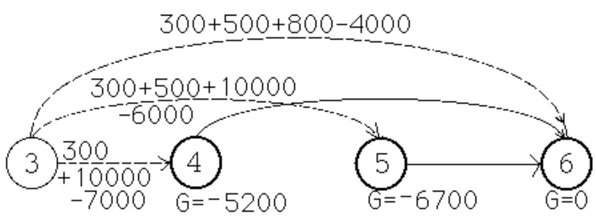
= minimum{ ¯3400, ¯5200 } = ¯5200

Stage 4

Computation of G[4] = Minimum total cost until end of time period 6, given a new machine at time 4

x	C	C+G
5	3300	¯3400
6	¯5200	¯5200

X*[4]= 6 = optimal replacement time
G[4]= ¯5200



G(3)= minimum{300+10000-7000+G(4),
300+500+10000-6000+G(5),
300+500+800-4000+G(6)}

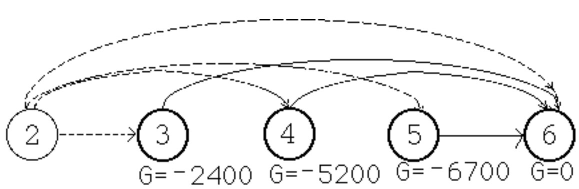
= minimum{ -1900, -1900, -2400 } = -2400

Stage 3

Computation of G[3] = Minimum total cost until end of time period 6, given a new machine at time 3

x	C	C+G
4	3300	¯1900
5	4800	¯1900
6	¯2400	¯2400

X*[3]= 6 = optimal replacement time
G[3]= ¯2400



G(2)= minimum{3300+G(3), 4800+G(4),
7600+G(5), ¯200+G(6)}
= minimum{ 900, ¯400, 900, ¯200} = ¯400

Stage 2

Computation of G[2] = Minimum total cost until end of time period 6, given a new machine at time 2

x	C	C+G
3	3300	900
4	4800	¯400
5	7600	900
6	¯200	¯200

X*[2]= 4 = optimal replacement time
G[2]= ¯400

Stage 1

Computation of G[1] = Minimum total cost until end of time period 6, given a new machine at time 1

x	C	C+G
2	3300	2900
3	4800	2400
4	7600	2400
5	9800	3100
6	2400	2400

X*[1]= 3 = optimal replacement time
G[1]= 2400

Actually, one could replace at x=3, 4, or 6!

Stage 0

Computation of G[0] = Minimum total cost until end of time period 6, given a new machine at time 0

x	C	C+G
1	3300	5700
2	4800	4400
3	7600	5200
4	9800	4600
5	12400	5700
6	5600	5600

X*[0]= 2 = optimal replacement time
G[0]= 4400

Summary

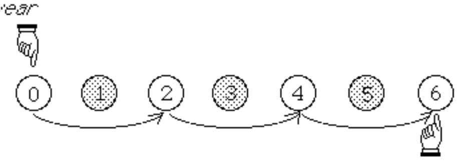
t	x	G
0	2	4400.00
1	3	2400.00
2	4	¯400.00
3	6	¯2400.00
4	6	¯5200.00
5	6	¯6700.00
6	0	0.00

Your expected total cost for 6 time periods will be 4400.00

The optimal plan is to replace the initial car after two years, i.e., X(0)=2.*
Then, since X(2)=4, you should replace at the end of the fourth year.*

Optimal replacement plan

Starting point:
New car at the beginning of the first year



Termination:
No car at the end of the sixth year