Optimal Batch Size

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What is the optimal batch size x which will minimize the expected cost of obtaining at least n good components?

A company purchases customized electronic components from a supplier.

- **n** = required number
- **x** = batch size ordered ($x \geq n$)

Each unit is inspected when received.

- **p** = probability that a component passes inspection

For each component passing inspection,

- **c** = unit price paid to supplier

If more than **n** components pass inspection,

- **v** = salvage value of excess components

If fewer than **n** components pass inspection,

- **K** = cost of re-ordering

Example

n = 20 components are required, each costing **c** = $50.

The probability that a component passes inspection is **p** = 80%.

The supplier charges **K** = $100 for the setup time, inconvenience, etc. of producing a new batch (in addition to the $50/acceptable component)

v = salvage value of excess components = $10

The number x of components which would yield an ***expected*** 20 acceptable components is ***n/p*** = 20/80% = 25.

However, ordering 25 will result in a significant probability that we must re-order, incurring the $100 ordering cost,
while ordering too large a batch will result in an excess of acceptable components, also incurring a large cost.

What is the batch size that will minimize the total expected cost?

Let

$p_x(j)$ = probability that j components of a batch of size **x** will pass inspection

Assuming independence of flaws,

$$p_x(j) = \frac{x!}{j!\,(x-j)!}\, p^j\, (1-p)^{x-j}$$

binomial distribution

Let **f(n)** = minimum expected cost of acquiring n acceptable components

We wish to compute f(20).

Recursive Definition of f

$$f(n) = \min_{x \geq n} \left\{ \sum_{j=0}^{x} j c\, p_x(j) - \sum_{j=n}^{x} (j-n) v\, p_x(j) + \sum_{j=0}^{n-1} \left[K + f(n-j)\right] p_x(j) \right\}$$

cost of acceptable parts — *salvage value of excess* — *expected cost when insufficient acceptable parts*

$$\ldots + \sum_{j=0}^{n-1} \left[K + f(n-j)\right] p_x(j)$$

Note that f(n) appears in these terms (when j=0)

$$f(n) = \min_{x \ge n} \left\{ \sum_{j=0}^{x} jcp_x(j) - \sum_{j=n}^{x} (j-n)vp_x(j) + \sum_{j=1}^{n-1} \left[K+f(n-j)\right]p_x(j) + \left[K+f(n)\right]p_x(0) \right\}$$

Denote the optimal x by $\hat{x}$

$$f(n) - p_{\hat{x}}(0)f(n) = \sum_{j=1}^{\hat{x}} jcp_{\hat{x}}(j) - \sum_{j=n}^{\hat{x}} (j-n)vp_{\hat{x}}(j) + \sum_{j=1}^{n-1} \left[K+f(n-j)\right]p_{\hat{x}}(j) + Kp_{\hat{x}}(0)$$

Solving for f(n) yields

$$f(n) = \min_{x \ge n} \left\{ \frac{c\sum_{j=0}^{x} jp_x(j) - v\sum_{j=n}^{x}(j-n)p_x(j) + K\sum_{j=0}^{n-1}p_x(j) + \sum_{j=1}^{n-1} f(n-j)p_x(j)}{1-p_x(0)} \right\}$$

which can be evaluated recursively, setting f(0)=0, computing f(1), f(2), f(3), ... f(n).


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Parameters for Optimal Batch Size Problem

   N = required number of acceptable components = 20
   C = cost of each component passing inspection = 50
   V = salvage value of excess components = 10
   K = cost of reordering = 100
   p = probability of passing inspection = 0.8
Xmax = maximum batch size considered = 30
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```
    ∇P←p BINOMIAL∆TABLE N;X;J
[1]   ⍝
[2]   ⍝         Compute table of binomial probabilities
[3]   ⍝
[4]   P←(N,N+1)⍴0
[5]   J←0    ⋄ X←1
[6]   NEXT∆X:J←J,X
[7]   P[X;J+1]←((!X)÷(!J)×(!X-J))×(p⋆J)×(1-p)⋆(X-J)
[8]   →NEXT∆X IF N≥X←X+1
    ∇
```

x \ j:	0	1	2	3	4	5	6	7	8
1	20000	80000	00000	00000	00000	00000	00000	00000	00000
2	04000	32000	64000	00000	00000	00000	00000	00000	00000
3	00800	09600	38400	51200	00000	00000	00000	00000	00000
4	00160	02560	15360	40960	40960	00000	00000	00000	00000
5	00032	00640	05120	20480	40960	32768	00000	00000	00000
6	00006	00154	01536	08192	24576	39322	26214	00000	00000
7	00001	00036	00430	02867	11469	27525	36700	20972	00000
8	00000	00008	00115	00918	04588	14680	29360	33554	16777

Binomial Distribution

Stage 1

x	________Expected Costs_____ Purchase	Salvage	Reorder	1-P0	Total ÷(1-P0)
2	80.00	¯6.40	4.00	0.9600	80.83

Example: Suppose 1 unit is need, and 2 are ordered...

$p_2(0)=0.04,\ p_2(1)=0.32,\ p_2(2)=0.64$

That is, there is probability 4% that neither of the 2 received parts pass inspection, and so the expected reorder cost will be 0.04 × $100 = $4

The expected number passing inspection will be 0x4% + 1x32% + 2x64% = 1.6, so expected purchase price is $50 × 1.6 = $80

Stage 1

x	________Expected Costs_____ Purchase	Salvage	Reorder	1-P0	Total ÷(1-P0)
2	80.00	¯6.40	4.00	0.9600	80.83

The expected number of excess parts will be 0 × 4% + 0 × 32% + 1 × 64% = 0.64 and so the expected salvage value is 6.4 × $10 = $6.4

Total is 80-6.4+4 = $77.6

Probability that at least one part passes inspection is 96%, and the quantity to be compared when minimizing is

$$\frac{\$77.6}{0.96} = \$80.83$$

Stage 1

x	________Expected Costs_____ Purchase	Salvage	Reorder	1-P0	Total ÷(1-P0)
1	40.00	0.00	20.00	0.8000	75.00
2	80.00	¯6.40	4.00	0.9600	80.83
3	120.00	¯14.08	0.80	0.9920	107.58

F[1] = 75 at X⋆ = 1

Doing a similar computation with order quantity x=1 and x=3 yields the values $75 and $107.56, respectively.

The optimal order quantity is therefore x=1, which yields a minimum expected cost of f(1) = $75 to obtain one acceptable part.

Once f(1) has been computed, then f(2) may be computed:

Stage 2

x	________Expected Costs_____ Purchase	Salvage	Reorder	1-P0	Total ÷(1-P0)
1	40.00	0.00	160.00	0.8000	250.00
2	80.00	0.00	60.00	0.9600	145.83
3	120.00	¯5.12	17.60	0.9920	133.55
4	160.00	¯12.29	4.64	0.9984	152.60
5	200.00	¯20.07	1.15	0.9997	181.14

F[2] = 133.55 at X⋆ = 3

That is, if 2 acceptable parts are required, it is optimal to place the order for 3 parts, and the expected cost is $133.55.

Next, given f(1) and f(2), the value of f(3) may be computed:

Stage 3

x	_______Expected Purchase	Costs_____ Salvage	Reorder	1-P0	Total ÷(1-P0)
3	120.00	0.00	90.42	0.9920	212.12
4	160.00	¯4.10	33.02	0.9984	189.23
5	200.00	¯10.65	10.49	0.9997	199.90
6	240.00	¯18.19	3.05	0.9999	224.88

F[3] = 189.23 at X* = 4

Stage 4

x	_______Expected Purchase	Costs_____ Salvage	Reorder	1-P0	Total ÷(1-P0)
4	160.00	0.00	115.12	0.9984	275.56
5	200.00	¯3.28	49.68	0.9997	246.48
6	240.00	¯9.18	18.37	0.9999	249.21
7	280.00	¯16.38	6.13	1.0000	269.75

F[4] = 246.48 at X* = 5

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Stage 5

x	_______Expected Purchase	Costs_____ Salvage	Reorder	1-P0	Total ÷(1-P0)
5	200.00	0.00	136.57	0.9997	336.68
6	240.00	¯2.62	67.12	0.9999	304.52
7	280.00	¯7.86	28.14	1.0000	300.28
8	320.00	¯14.68	10.53	1.0000	315.85
9	360.00	¯22.23	3.62	1.0000	341.40

F[5] = 300.28 at X* = 7

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...etc.

The optimal batch size happens to be 25!

(i.e., the batch size which yields an expected 20 good components)

If you require	Then order	Expected cost
20	25	1082.20
19	24	1030.50
18	23	979.22
17	22	928.44
16	20	876.67
15	19	824.39
14	18	772.60
13	17	721.40
12	15	669.89
11	14	616.78
10	13	564.27
9	12	512.52
8	10	460.87
7	9	406.41
6	8	352.78
5	7	300.28
4	5	246.48
3	4	189.23
2	3	133.55
1	1	75.00

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