

What is the optimal batch size **x** which will minimize the expected cost of obtaining at least **n** good components?

The number x of components which would yield an *expected* 20 acceptable components is n/p = 20/80% = 25.

However, ordering 25 will result in a significant probability that we must re-order, incurring the \$100 ordering cost, while ordering too large a batch will result

in an excess of acceptable components, also incurring a large cost.

## What is the batch size that will minimize the total expected cost?

©Dennis Bricker, U. of Iowa, 1997

Let **f**(**n**) = minimum expected cost of acquiring n acceptable components We wish to compute f(20).

Recursive Definition of f  $\sum j c p_x(j)$  $f(n) = min \downarrow$  $(j-n)\mathbf{v}\mathbf{p}_{\mathbf{X}}(j) +$  $\sum |\mathbf{K} + \mathbf{f}(\mathbf{n} - \mathbf{j})| \mathbf{p}_{\mathbf{x}}(\mathbf{j})$ cost of sa/vade expected cost acceptable value of when insufficient parts excess acceptable parts

A company purchases customized electronic components from a supplier.

- **n** = required number
- x = batch size ordered (x≥n)

Each unit is inspected when received.

- p = probability that a component passes inspection
- For each component passing inspection,
- **c** = unit price paid to supplier
- If more than **n** components pass inspection, **v** = salvage value of excess components
- If fewer than **n** components pass inspection, K = cost of re-ordering

©Dennis Bricker, U. of Iowa, 1997

## Example

- n = 20 components are required, each costing
  c = \$50.
- The probability that a component passes inspection is  $\mathbf{p} = 80\%$ .
- The supplier charges K = \$100 for the setup time, inconvenience, etc. of producing a new batch (in addition to the \$50/acceptable component)
- v = salvage value of excess components = \$10

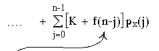
©Dennis Bricker, U. of Iowa, 1997

Let

Assuming independence of flaws,

 $p_{x}(j) = \frac{x!}{j! (x-j)!} p^{j} (1-p)^{x-j}$  binomial distribution

©Dennis Bricker, U. of Iowa, 1997



<sup>(</sup>Note that f(n) appears in these terms (when j=0)



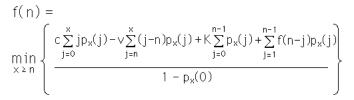
$$\begin{split} f(n) &= \min_{x \ge n} \left\{ \sum_{j=0}^{x} j c p_{x}(j) - \sum_{j=n}^{x} (j-n) v p_{x}(j) \right. \\ &+ \sum_{j=1}^{n-1} [K + f(n-j)] p_{x}(j) + [K + f(n)] p_{x}(0) \right\} \end{split}$$

Denote the optimal x by 
$$\hat{x}$$
  
 $f(n) - p_{\hat{x}}(0)f(n) = \sum_{j=1}^{\hat{x}} jcp_{\hat{x}}(j) - \sum_{j=n}^{\hat{x}} (j-n)vp_{\hat{x}}(j)$   
 $+ \sum_{j=1}^{n-1} [K+f(n-j)]p_{\hat{x}}(j) + Kp_{\hat{x}}(0)$ 

©Dennis Bricker, U. of Iowa, 1997

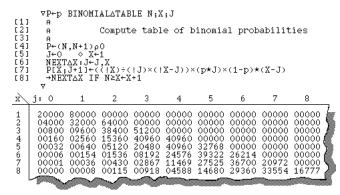
©Dennis Bricker, U. of Iowa, 1997

Solving for f(n) yields



which can be evaluated recursively, setting f(0)=0, computing f(1), f(2), f(3), ... f(n).

©Dennis Bricker, U. of Iowa, 1997



Binomial Distribution

©Dennis Bricker, U. of Iowa, 1997

Stage 1								
x		xpected Co Salvage	osts Reorder	1-P0	Total ÷(1-PO)			
2	80.00	-6.40	4.00	0.9600	80.83			

The expected number of excess parts will be

 $0 \times 4\% + 0 \times 32\% + 1 \times 64\% = 0.64$  and so the expected salvage value is  $6.4 \times $10 = $6.4$ 

Total is 80-6.4+4 = \$77.6

Probability that at least one part passes inspection is 96%, and the quantity to be compared when minimizing is

$$\frac{\$77.6}{0.96}$$
 = \\$80.83

©Dennis Bricker, U. of Iowa, 1997

Once f(1) has been computed, then f(2) may be computed:

x		xpected Co Salvage	osts Reorder	1-P0	Total ÷(1-P0)
1 2 3 4 5	40.00 80.00 120.00 160.00 200.00	0.00 0.00 -5.12 -12.29 -20.07	$160.00 \\ 60.00 \\ 17.60 \\ 4.64 \\ 1.15$	0.8000 0.9600 0.9920 0.9984 0.9997	250.00 145.83 133.55 152.60 181.14

F[2] = 133.55 at  $X \star = 3$ 

That is, if 2 acceptable parts are required, it is optimal to place the order for 3 parts, and the expected cost is \$133.55.

Parameters for Optimal Batch Size Problem

- N = required number of acceptable components = 20
- = cost of each component passing inspection = 50 = salvage value of excess components = 10 C
- v
- K = cost of reordering = 100
- = probability of passing inspection = 0.8 p
- Xmax = maximum batch size considered = 30

		Stag			
x		xpected Cos Salvage R	ts eorder	1-P0	Total ÷(1-P0)
2	80.00	-6.40	4.00	0.9600	80.83

Example: Suppose 1 unit is need, and 2 are ordered...

## $p_2(0)=0.04, p_2(1)=0.32, p_2(2)=0.64$

That is, there is probability 4% that neither of the 2 received parts pass inspection, and so the expected reorder cost will be 0.04 × \$100 = \$4

The expected number passing inspection will be

 $0\times4\% + 1\times32\% + 2\times64\% = 1.6$ , so expected purchase price is  $50 \times 1.6 = 80$ 

©Dennis Bricker, U. of Iowa, 1997



	E	xpected Co		Total	
x	Purchase	Salvage	Reorder	1-P0	÷(1-PO)
1	40.00	0.00	20.00	0.8000	75.00
2	80.00 120.00	-6.40 -14.08	4.00 0.80	0.9600 0.9920	80.83

 $F[1] = 75 \text{ at } X \star = 1$ 

Doing a similar computation with order quantity x=1 and x=3yields the values \$75 and \$107.56, respectively.

The optimal order quantity is therefore x=1, which yields a minimum expected cost of f(1) = \$75 to obtain one acceptable part.

## Next, given f(1) and f(2), the value of f(3) may be computed:

	Stage 3									
x	E Purchase	xpected Co Salvage	osts Reorder	1-P0	Total ÷(1-P0)					
3 4 5 6	120.00 160.00 200.00 240.00	0.00 -4.10 -10.65 -18.19	90.42 33.02 10.49 3.05	0.9920 0.9984 0.9997 0.9999	212.12 189.23 199.90 224.88					

F[3] = 189.23 at X\* = 4

©Dennis Bricker, U. of Iowa, 1997

Stage 5

x	E Purchase	xpected C Salvage	osts Reorder	1-P0	Total ÷(1-P0)
5	200.00	0.00	136.57	0.9997	336.68
6	240.00	-2.62	67.12	0.9999	304.52
7	280.00	-7.86	28.14	1.0000	300.28
8	320.00	-14.68	10.53	1.0000	315.85
9	360.00	-22.23	3.62	1.0000	341.40

F[5] = 300.28 at X\* = 7

©Dennis Bricker, U. of Iowa, 1997

	.etc.	

The optimal batch size happens to be 25!

(i.e., the batch size which yields an expected 20 good components)

KÞ

$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	If you	Then	Expected
	require	order	cost
3 4 189.23 2 3 133.55 1 1 75.00	20 19 18 17 16 15 14 13 12 11 10 9	25 24 22 20 18 15 14 15 14 12 10 8 7 5 4 3	$\begin{array}{c} 1082.20\\ 1030.50\\ 979.22\\ 928.44\\ 876.67\\ 824.39\\ 772.60\\ 721.40\\ 669.89\\ 616.78\\ 564.27\\ 512.52\\ 460.87\\ 406.41\\ 352.78\\ 352.78\\ 352.78\\ 352.78\\ 352.55\\ 133.55\\ \end{array}$

©Dennis Bricker, U. of Iowa, 1997



	Е	xpected (	Costs		Total
x	Purchase	Salvage	Reorder	1-P0	÷(1-P0)
4 5 6 7	160.00 200.00 240.00 280.00	0.00 -3.28 -9.18 -16.38	115.12 49.68 18.37 6.13	0.9984 0.9997 0.9999 1.0000	275.56 246.48 249.21 269.75

F[4] = 246.48 at X\* = 5

©Dennis Bricker, U. of Iowa, 1997