Branch-&-Bound Algorithm for the Asymmetric TSP

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$$\text{Minimize} \sum_{i=1}^{n} \sum_{j=1}^{n} d_{ij} X_{ij}$$

subject to

$$\left.\begin{aligned} \sum_{i=1}^{n} X_{ij} &= 1 \quad \forall\, j=1,\ldots n \\ \sum_{j=1}^{n} X_{ij} &= 1 \quad \forall\, i=1,\ldots n \end{aligned}\right\} \text{Assignment constraints}$$

$$X_{ij} \in \{0,1\} \quad \forall\ i,j$$

plus subtour elimination constraints

Relaxing the subtour elimination constraints, we are left with an assignment problem, whose solution provides us with a ***lower bound*** on the length of the optimal tour!

$$\text{Minimize} \sum_{i=1}^{n} \sum_{j=1}^{n} d_{ij} X_{ij}$$

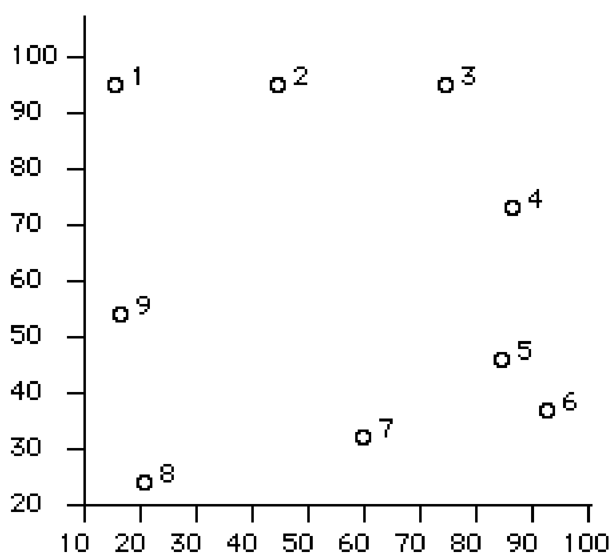
subject to

$$\sum_{i=1}^{n} X_{ij} = 1 \ \forall \ j=1,\ldots n$$

$$\sum_{j=1}^{n} X_{ij} = 1 \ \forall \ i=1,\ldots n$$

$$X_{ij} \in \{0,1\} \ \forall \ i,j$$

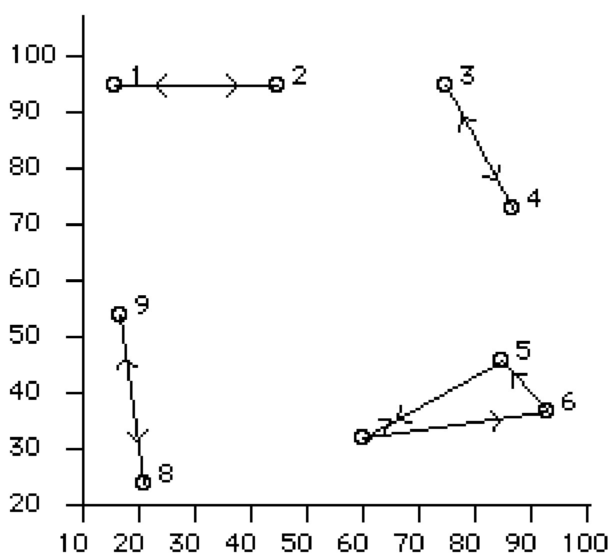
Example



Distances

from \ to	1	2	3	4	5	6	7	8	9
1	0	36	64	68	93	104	68	68	39
2	24	0	35	41	71	83	56	72	48
3	57	28	0	29	48	52	73	95	81
4	69	54	30	0	35	44	40	79	71
5	80	70	55	21	0	20	20	65	66
6	91	82	66	30	20	0	24	70	76
7	75	63	74	53	27	24	0	46	58
8	69	73	98	86	66	64	48	0	40
9	39	48	80	77	66	69	56	36	0

Solution of Assignment Problem



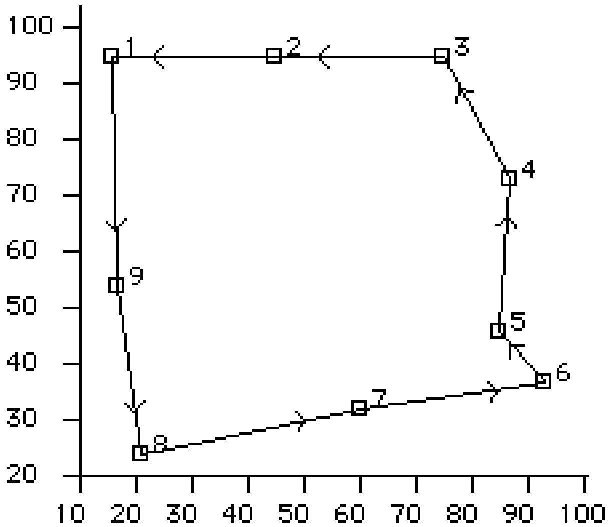
Minimum Assignment Cost = 259

Optimal assignments (i→j)

i=	1	2	3	4	5	6	7	8	9
j=	2	1	4	3	7	5	6	9	8

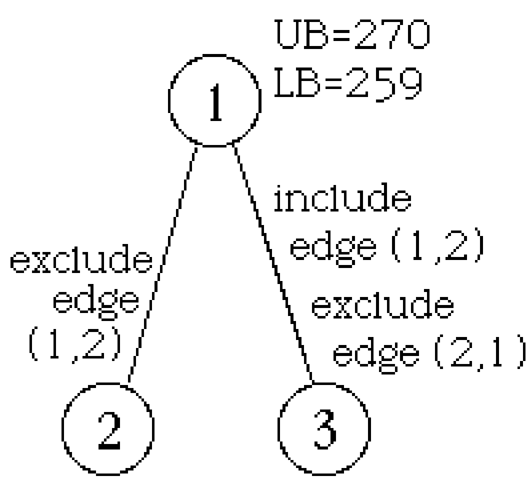
Applying Heuristic Algorithm

>>>New incumbent has been found, with length 270
Tour= 1 9 8 7 6 5 4 3 2 1



Solution of Asymmetric TSP by Branch-&-Bound

Choose a subtour in the AP solution: 1 → 2 → 1



Subproblem number 2 (level 1)

Edges excluded

1
2

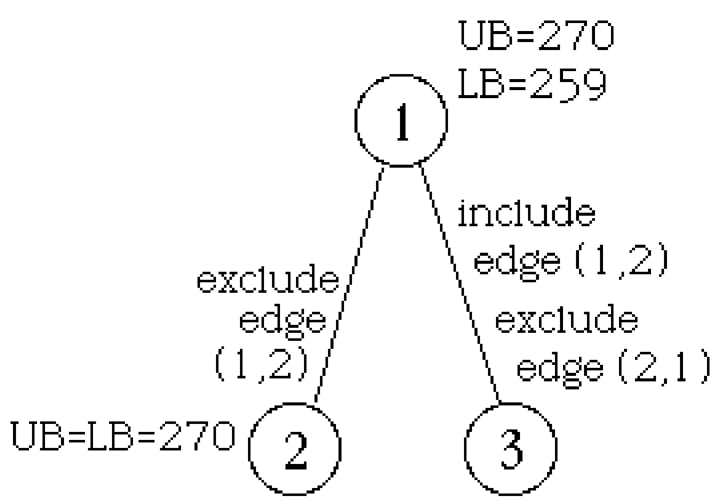
Minimum Assignment Cost = 270 (≥ incumbent = 270)

Optimal assignments (i→j)

i=	1	2	3	4	5	6	7	8	9
j=	9	1	2	3	4	5	6	7	8

Tour!

(same as incumbent found by the heuristic algorithm)



Subproblem number 3 (level 2)

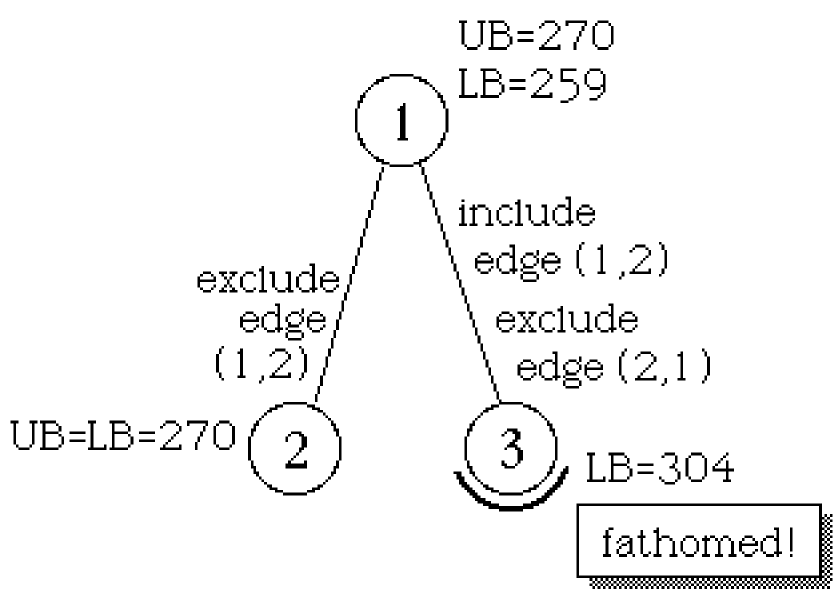
Edges included	Edges excluded
1	2
2	1

Minimum Assignment Cost = 304 (≥ incumbent = 270)

Optimal assignments (i→j)

i=	1	2	3	4	5	6	7	8	9
j=	2	4	1	3	7	5	6	9	8

Not a tour, but Lower Bound (304) exceeds incumbent!



The incumbent must be optimal!