Eliminating sets from consideration

Suppose that P_j is a lower bound on the cost of all feasible solutions which include set j.

If P_j exceeds the cost of a known feasible solution, then set j can be eliminated from any further consideration... it cannot be included in any optimal solution.

How can we compute such a lower bound?

author

Consider a modification of our original problem, in which the constraint $X_j=1$ is added.

Consider further the Lagrangian relaxation of that problem (in which all but the constraint $X_j=1$ is relaxed).

If we use the same values of the Lagrangian multipliers as those used in the relaxation of the original problem, the Lagrangian relaxation of the modified problem is easily solved:

If we add the restriction $X_j=1$ to the set covering problem, then the optimal solution of the Lagrangian relaxation (with the added restriction) will be

$$P_j = \begin{cases} \Phi(\lambda) + \underbrace{\bar{C}_j}_{\text{reduced cost of } X_j} & \text{if } X_j=0 \text{ in the solution of the current Lagrangian relaxation} \\ \Phi(\lambda) & \text{if } X_j=1 \text{ in the solution of the current Lagrangian relaxation} \end{cases}$$

This gives us a lower bound on the cost of all feasible solutions of the set covering problem which contain set j.

Heuristic Adjustment Procedures

Each time we evaluate $\Phi(\lambda)$, i.e., solve a Lagrangian relaxation, we obtain a solution which in general leaves some points uncovered and/or covers some points more than once.

By adding &/or removing sets, we can perhaps obtain a good solution of the original problem.

If any points remain uncovered, our heuristic adjustment procedure must

- select the next point to be covered
- select one of the sets covering this point

After all points are covered, any superfluous set (whose removal does not leave any point uncovered) is removed from the solution.

Selecting an uncovered point

- select one of the uncovered points at random
- select the uncovered point having the largest Lagrangian multiplier
- select the point having the smallest number of potential covering sets

Selecting a set to be added:

- Add the set which covers the point at lowest cost (Beasley)
- Consider as candidates the k least-cost sets covering the point; add the set which has the lowest "reduced cost".
- Consider as candidates the k least-cost sets covering the point; recompute the "reduced cost" with zero assigned to Lagrangian multipliers of points already covered.