

# Set Covering Problem

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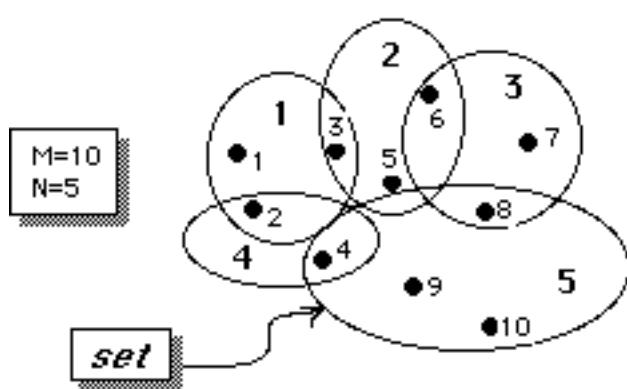


- ☒ Problem definition & formulation
- ☒ Applications
- ☒ Lagrangian relaxation
- ☒ Lagrangian dual problem
- ☒ Solving dual by subgradient method
- ☒ Solving dual by dual ascent method
- ☒ Heuristic based on Lagrangian relaxation
- ☒ Eliminating sets during Lagrangian relaxation
- ☒ Example: subgradient optimization
- ☒ Example: dual ascent
- ☒ Computational results

Given  $M$  points, and  $N$  sets each containing one or more points:

Let  $C_j$  = cost of set  $\#j$

$$a_{ij} = \begin{cases} 1 & \text{if point } i \text{ is an element of set } j \\ 0 & \text{otherwise} \end{cases}$$



$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Define variables  $X_j = \begin{cases} 1 & \text{if set } \#j \text{ is selected} \\ 0 & \text{otherwise} \end{cases}$

Set Covering Problem

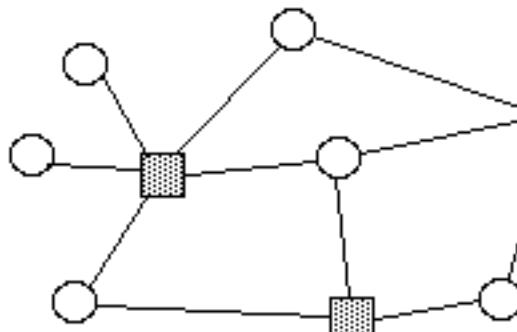
$$\text{Minimize } \sum_{j=1}^N C_j X_j$$

$$\text{subject to } \sum_{j=1}^N a_{ij} X_j \geq 1 \quad \text{for each } i=1, 2, \dots, M$$

$$X_j \in \{0, 1\} \quad \text{for each } j=1, 2, \dots, N$$

## Applications

### Facility Location



- = potential facility site
- = customer location
- $C_j$  = cost of building facility  $j$

*Facilities must be selected to serve every customer. Possible links are indicated.*

*(customers are "points", facilities are "sets")*

## Applications

### Information Retrieval

Retrieve a given set of  $m$  requests for information from a set of  $n$  files so that the length of the search is minimized.

$C_j$  = length of file  $j$

$a_{ij} = 1$  if the  $i^{\text{th}}$  information requested is in file  $j$ ,  
0 otherwise

*(information requests are "points", and the files are the "sets")*