

Set
Covering
Problem



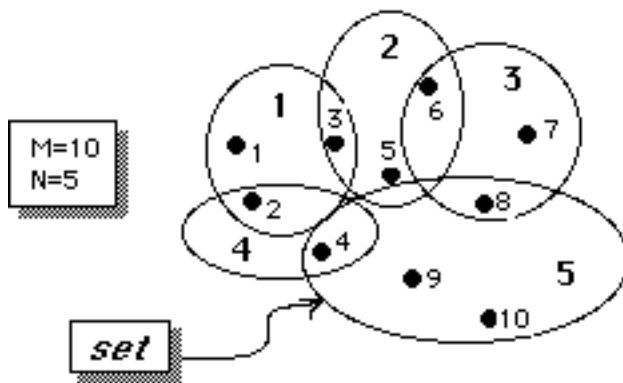
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- ☞ Problem definition & formulation
- ☞ Applications
- ☞ Lagrangian relaxation
- ☞ Lagrangian dual problem
- ☞ Solving dual by subgradient method
- ☞ Solving dual by dual ascent method
- ☞ Heuristic based on Lagrangian relaxation
- ☞ Eliminating sets during Lagrangian relaxation
- ☞ Example: subgradient optimization
- ☞ Example: dual ascent
- ☞ Computational results

Given M points, and N sets each containing one or more points:

Let C_j = cost of set # j

$$a_{ij} = \begin{cases} 1 & \text{if point } i \text{ is an element of set } j \\ 0 & \text{otherwise} \end{cases}$$



$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Define variables $X_j = \begin{cases} 1 & \text{if set } \#j \text{ is selected} \\ 0 & \text{otherwise} \end{cases}$

Set Covering Problem

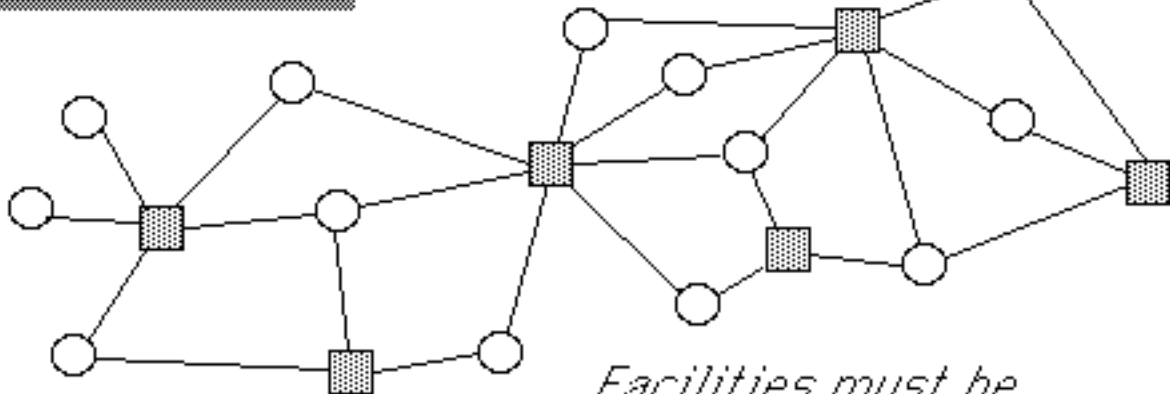
$$\text{Minimize } \sum_{j=1}^N C_j X_j$$

$$\text{subject to } \sum_{j=1}^N a_{ij} X_j \geq 1 \quad \text{for each } i=1, 2, \dots, M$$

$$X_j \in \{0,1\} \quad \text{for each } j=1, 2, \dots, N$$

Applications

Facility Location



- = potential facility site
- = customer location
- C_j = cost of building facility j

Facilities must be selected to serve every customer. Possible links are indicated.

(customers are "points", facilities are "sets")

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Applications

Information Retrieval

Retrieve a given set of m requests for information from a set of n files so that the length of the search is minimized.

C_j = length of file j

$a_{ij} = 1$ if the i^{th} information requested is in file j ,
0 otherwise

(information requests are "points", and the files are the "sets")