Batch Processing Problem

Markov Decision Model

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Each minute, either zero, one, or two customers arrive at a facility which has a capacity (batch size) of 5.

i=#waiting	0	1	2	3	4
P{0 arrivals}	0.04	0.25	0.44	0.61	0.76
P{1 arrival}	0.6	0.5	0.4	0.3	0.2
P{2 arrivals}	0.36	0.25	0.16	0.09	0.04

(Probability of arrival diminishes as the queue lengthens... probability of 2 arrivals is square of probability of one.)

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There is a "holding cost" per customer of 10¢ per minute, and a processing cost of 40¢ per batch *(independent of batch size)*.

What is the optimal queue length for batch processing?

(The smaller the batches, the more the processing cost, while the larger the batches, the more the holding cost.)

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Markov Decision Model

States

i	name
1	0 waiting
2	1 waiting
3	2 waiting
4	3 waiting
5	4 waiting
6	≥5 waiting

Actions

k	name
1	wait another minute
2	process the batch

state defined as number in the queue at the beginning of the stage (minute) before any arrivals

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Cost Matrix

name	1	2	3	4	5	6
wait another minute	0	0.1	0.2	0.3	0.4	999
process batch	0.4	0.5	0.6	0.7	0.8	0.9

includes holding cost for customers in queue at beginning of stage

A value of 999 above signals
an infeasible action in a state.

(Rows ~ actions, Columns ~ states)

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Transition Probabilities

Action: wait another minute

from \ to	1	2	3	4	5	6
1	0.04	0.6	0.36	0	0	0
2	0	0.25	0.5	0.25	0	0
3	0	0	0.44	0.4	0.16	0
4	0	0	0	0.61	0.3	0.09
5	0	0	0	0	0.76	0.24
	0	0	0	0	0	1

Action: process the batch

from \ to	1	2	3	4	5	6
1	0.04	0.6	0.36	0	0	0
2	0.04	0.6	0.36	0	0	0
3	0.04	0.6	0.36	0	0	0
4	0.04	0.6	0.36	0	0	0
5	0.04	0.6	0.36	0	0	0
	0.04	0.6	0.36	0	0	0

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LP Tableau

k	1	2	1	2	1	2	1	2	1	2	2	RHS
i	1	1	2	2	3	3	4	4	5	5	6	
	0	0.4	0.1	0.5	0.2	0.6	0.3	0.7	0.4	0.8	0.9	
	0.96	0.96	0	¯0.04	0	¯0.04	0	¯0.04	0	¯0.04	¯0.04	0
	¯0.6	¯0.6	0.75	0.4	0	¯0.6	0	¯0.6	0	¯0.6	¯0.6	0
	¯0.36	¯0.36	¯0.5	¯0.36	0.56	0.64	0	¯0.36	0	¯0.36	¯0.36	0
	0	0	¯0.25	0	¯0.4	0	0.39	1	0	0	0	0
	0	0	0	0	¯0.16	0	¯0.3	0	0.24	1	0	0
	1	1	1	1	1	1	1	1	1	1	1	1

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initial basic solution corresponds to the policy:
wait until 5 customers arrive
before processing a batch

	☆		☆		☆		☆		☆		☆	
k:	1	2	1	2	1	2	1	2	1	2	2	
i:	1	1	2	2	3	3	4	4	5	5	6	rhs
Min	0	0.4	0	0.2180	0	0.0380	0	¯0.1080	0	¯0.2187	0	¯0.3715
	1	1	0	¯0.0382	0	¯0.0340	0	¯0.0276	0	¯0.0199	0	0.0047
	0	0	1	0.5678	0	¯0.6801	0	¯0.5528	0	¯0.3993	0	0.0958
	0	0	0	¯0.1081	1	0.6318	0	¯0.9379	0	¯0.6774	0	0.1625
	0	0	0	0.2530	0	0.2120	1	1.2476	0	¯0.9507	0	0.2281
	0	0	0	0.2441	0	0.6863	0	0.9342	1	2.5266	0	0.3936
	0	0	0	0.0813	0	0.1838	0	0.3365	0	0.5208	1	0.1150

Iteration 0

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	☆		☆		☆		☆		☆		☆	
k:	1	2	1	2	1	2	1	2	1	2	2	
i:	1	1	2	2	3	3	4	4	5	5	6	rhs
Min	0	0.4	0	0.2180	0	0.0380	0	¯0.1080	0	¯0.2187	0	¯0.3715
	1	1	0	¯0.0382	0	¯0.0340	0	¯0.0276	0	¯0.0199	0	0.0047
	0	0	1	0.5678	0	¯0.6801	0	¯0.5528	0	¯0.3993	0	0.0958
	0	0	0	¯0.1081	1	0.6318	0	¯0.9379	0	¯0.6774	0	0.1625
	0	0	0	0.2530	0	0.2120	1	1.2476	0	¯0.9507	0	0.2281
	0	0	0	0.2441	0	0.6863	0	0.9342	1	(2.5266)	0	0.3936
	0	0	0	0.0813	0	0.1838	0	0.3365	0	0.5208	1	0.1150

Iteration 0

↑ *pivot column*

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initial basic solution corresponds to the policy:
wait until 5 customers arrive
before processing a batch

Iteration 0

Policy: (Cost= 0.3715)

steadystate distribution ↓

	State	Action	P{i}
1	0 waiting	1 wait another minute	0.00479179
2	1 waiting	1 wait another minute	0.0958357
3	2 waiting	1 wait another minute	0.162578
4	3 waiting	1 wait another minute	0.22818
5	4 waiting	1 wait another minute	0.393611
6	≥5 waiting	2 process batch	0.115003

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Iteration 1

	★		★		★		★			★	★	
k:	1	2	1	2	1	2	1	2	1	2	2	
i:	1	1	2	2	3	3	4	4	5	5	6	rhs
Min	0	0.4	0	0.2391	0	0.0974	0	¯0.0271	0.0865	0	0	¯0.3374
	1	1	0	¯0.0363	0	¯0.0285	0	¯0.0202	0.0079	0	0	0.0079
	0	0	1	0.6064	0	¯0.5716	0	¯0.4052	0.1580	0	0	0.1580
	0	0	0	¯0.0426	1	0.8159	0	¯0.6874	0.2681	0	0	0.2681
	0	0	0	0.3449	0	0.4703	1	1.5992	0.3762	0	0	0.3762
	0	0	0	0.0966	0	0.2716	0	0.3697	0.3957	1	0	0.1557
	0	0	0	0.0310	0	0.0423	0	0.1439	¯0.2061	0	1	0.0338

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Iteration 1

	★		★		★		★			★	★	
k:	1	2	1	2	1	2	1	2	1	2	2	
i:	1	1	2	2	3	3	4	4	5	5	6	rhs
Min	0	0.4	0	0.2391	0	0.0974	0	¯0.0271	0.0865	0	0	¯0.3374
	1	1	0	¯0.0363	0	¯0.0285	0	¯0.0202	0.0079	0	0	0.0079
	0	0	1	0.6064	0	¯0.5716	0	¯0.4052	0.1580	0	0	0.1580
	0	0	0	¯0.0426	1	0.8159	0	¯0.6874	0.2681	0	0	0.2681
	0	0	0	0.3449	0	0.4703	1	(1.5992)	0.3762	0	0	0.3762
	0	0	0	0.0966	0	0.2716	0	0.3697	0.3957	1	0	0.1557
	0	0	0	0.0310	0	0.0423	0	0.1439	¯0.2061	0	1	0.0338

↑ *pivot column*

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Iteration 1

Policy: (Cost= 0.337422)

	State	Action	P{i}
1	0 waiting	1 wait another minute	0.00790216
2	1 waiting	1 wait another minute	0.158043
3	2 waiting	1 wait another minute	0.268109
4	3 waiting	1 wait another minute	0.376294
5	4 waiting	2 process batch	0.155786
6	≥5 waiting	2 process batch	0.0338664

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Iteration 2

	☆		☆		☆			☆		☆	☆	
k:	1	2	1	2	1	2	1	2	1	2	2	
i:	1	1	2	2	3	3	4	4	5	5	6	rhs
Min	0	0.4	0	0.2450	0	0.1054	0.0169	0	0.0929	0	0	¯0.33104
	1	1	0	¯0.0319	0	¯0.0226	0.0126	0	0.0126	0	0	0.01266
	0	0	1	0.6938	0	¯0.4524	0.2533	0	0.2533	0	0	0.25339
	0	0	0	0.1055	1	1.0181	0.4298	0	0.4298	0	0	0.42986
	0	0	0	0.2156	0	0.2941	0.6252	1	0.2352	0	0	0.23529
	0	0	0	0.0168	0	0.1628	¯0.2312	0	0.3087	1	0	0.06877
	0	0	0	0	0	0	¯0.09	0	¯0.24	0	1	0

Reduced costs are nonnegative!

Optimal Policy!

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Iteration 2

Policy: (Cost= 0.331041)

	State	Action	P{i}
1	0 waiting	1 wait another minute	0.0126697
2	1 waiting	1 wait another minute	0.253394
3	2 waiting	1 wait another minute	0.429864
4	3 waiting	2 process batch	0.235294
5	4 waiting	2 process batch	0.0687783
6	≥5 waiting	2 process batch	0

Optimal Policy!

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