



# Interest Factors (time value of money)



## Single-payment factors

Let  $P$  = original investment

$r$  = rate of interest per period

$n$  = number of periods

$S_n$  = value of investment after  $n$  periods

Then  $S_n = (1 + r)^n P$

$\text{spcaf}(r,n) = (1 + r)^n$

*single-payment  
compound-amount factor*

Conversely, a future payment of  $S_n$  has an equivalent present worth  $P$ ,

$$P = (1 + r)^{-n} S_n$$

$$\text{sppwf}(r,n) = (1 + r)^{-n}$$

*single-payment  
present - worth  
factor*

## Uniform Series of Payments

Consider a sequence of  $n$  uniform periodic payments,  $R$ , earning interest at rate  $r$  per period, compounded at the end of each period. Then the accumulated value after  $n$  periods is

$$S_n = \frac{(1 + r)^n - 1}{r} R$$

$$\text{uscaf}(r,n) = \frac{(1 + r)^n - 1}{r}$$

*uniform - series  
compound - amount  
factor*

Conversely,  
the amount of each payment  $R$  required to accumulate  
a sum  $S$  after  $n$  periods at interest rate  $r$  is

$$R = \frac{r}{(1+r)^n - 1} S_n$$

$$\text{sfd}(r,n) = \frac{r}{(1+r)^n - 1}$$

*sinking - fund  
deposit factor*

The sequence of  $n$  uniform payments,  $R$ , can also be  
expressed as a present worth  $P$ :

$$P = \frac{(1+r)^n - 1}{r(1+r)^n} R$$

$$\text{uspwf}(r,n) = \frac{(1+r)^n - 1}{r(1+r)^n}$$

*uniform - series  
present - worth  
factor*

Finally, expressing a present amount  $P$  as an equivalent sequence of  $n$  uniform payments  $R$  gives

$$R = \frac{r(1+r)^n}{(1+r)^n - 1} P$$

$$\text{crf}(r,n) = \frac{r(1+r)^n}{(1+r)^n - 1}$$

*capital  
recovery  
factor*

### Summary:

Given	Find	by multiplying with the
$P$	$S_n$	single-payment compound-amount factor (spcaf)
$S_n$	$P$	single-payment present-worth factor (sppwf)
$R$	$S_n$	uniform-series compound-amount factor (uscaf)
$S_n$	$R$	sinking-fund deposit factor (sdfd)
$R$	$P$	uniform-series present-worth factor (uspwf)
$P$	$R$	capital-recovery factor (crf)