Cargo Ship Fleet Planning via GP

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An international corporation has decided to let a 20-year contract for the transport of 4 million tons/year of bauxite and iron ore from a South American port to a U.S. port 3000 miles distant, and for the shipment of an equal tonnage of coal and other materials from the U.S. port back to the South American port.

Due to the magnitude of the project, the president of your shipping company has decided to make a bid and, if successful in winning the contract, to buy a new fleet of cargo ships for this job.

The president would like to know the minimum project cost for bidding purposes.

The optimal number of ships,
load tonnage capacity, and
cruising speed
should also be determined.

COSTS

- initial acquisition costs
  - ship
  - power plant
- fuel & other operating costs

(Rough) cost estimates:

Cost of ship (exclusive of power plant): $400/T.
Cost of power plant: $150/shaft-hp
Cost of fuel: $0.003/shaft-hp-hr

"Rules of Thumb"

- weight of cargo ship (exclusive of power plant)
  = cargo tonnage
- ("Admiralty Formula"):

$$\text{Shaft-hp.} = (\text{displacement tonnage})^{2/3} \frac{(\text{Velocity})^3}{K}$$

where unit of velocity is knots, and
$K = 400$.

Define variables

X_1 = # of ships
X_2 = cargo load capacity (tons)
X_3 = ship velocity (knots)
X_4 = fraction of time en route

Acquisition Costs

- Cost of ship (minus power plant) = \$400/ton & weight of ship = 50% of cargo load capacity

$$400 \times \frac{1}{2} X_2 = 200\, X_2$$

Acquisition Costs

- Cost of power plant = \$150/shaft-hp

&

$$\text{shaft-hp} = (\text{displacement tonnage})^{2/3} \frac{(\text{Velocity})^3}{400}$$

$$\text{Displacement tonnage} = \text{ship weight} + \text{cargo weight}$$
$$= \frac{1}{2} X_2 + X_2 = \frac{3}{2} X_2$$

Cost: $$150 \left(\frac{3}{2} X_2\right)^{2/3} \times \frac{X_3^3}{400} = 0.491389\, X_2^{2/3} X_3^3$$

Fuel Cost

\$0.003/shaft-hp hr.

$$0.003 \times \frac{\left(\frac{3}{2} X_2\right)^{2/3} X_3^3}{400} = 9.03\times10^{-6} X_2^{2/3} X_3^3 \quad \text{per hr.}$$

\# hrs/year = 8750

\# hrs enroute/year = $8750\, X_4$

Cost: $9.83\times10^{-6} X_2^{2/3} X \times 8750\, X_4$

$$= 0.086\, X_2^{2/3} X_3^3 X_4 \quad \text{per year, per ship}$$

Present value of total fuel costs for next 20 years

Using interest rate of 10% per year, and assuming that fuel is purchased annually, the uniform-series present worth factor is 8.514

Present value of future fuel cost is

$$0.732\, X_2^{2/3} X_3^3 X_4 \quad \text{per ship}$$

Total present value of costs:

$$0.491389X_1X_2^{2/3}X_3^3 + 200\,X_1X_2 + 0.732X_1X_2^{2/3}X_3^3X_4$$

power plants *ships* *fuel*

Constraints

- 4 million tons/yr are to be transported in each direction.
- Because of limited docking facilities, at most 1000 tons/hr may be loaded or unloaded.

Constraints

- 4 million tons/yr are to be transported in each direction:

For each ship,

$$\frac{\#\text{ round trips}}{\text{year}} = \frac{X_3 \text{ mi/hr}}{6000 \text{ mi/trip}} \bullet \frac{8750 \text{ hrs}}{\text{yr}} \bullet X_4$$

$$= \frac{8750}{6000} X_3 X_4 = 1.458 \; X_3 X_4 \text{ trips/yr}$$

Capacity per trip (each direction): X_2 tons/trip

# of ships in fleet = X_3

Total capacity (tons/yr): $1.458\, X_1 X_2 X_3\, X_4$ *(each direction)*

$$1.458\, X_1 X_2 X_3\, X_4 \geq 4 \times 10^6$$

$$\Rightarrow \quad \boxed{2.74 \times 10^6\, X_1^{-1} X_2^{-1} X_3^{-1}\; X_4^{-1} \;\leq 1}$$

Constraints

- Because of limited docking facilities,
  at most 1000 tons/hr may be loaded or unloaded.

Time required per (one-way) trip (in hrs):

en route: $\dfrac{3000 \text{ mi.}}{X_3 \text{ mi/hr}}$

unloading: $\dfrac{X_2 \text{ tons}}{1000 \text{ tons/hr.}}$

loading: $\dfrac{X_2 \text{ tons}}{1000 \text{ tons/hr.}}$

Total time per one-way trip $3000\, X_3^{-1} + 2 \times \dfrac{X_2}{1000}$

travel time per one-way trip

fraction of time ship spends en route

$$X_4 = \frac{3000\, X_3^{-1}}{3000\, X_3^{-1} + 2 \times \dfrac{X_2}{1000}}$$

travel + time in port per one-way trip

If we replace "=" with "≤", the constraint will be tight at the optimum!

$$X_4 \le \frac{3000\,X_3^{-1}}{3000\,X_3^{-1} + 2\times \frac{X_2}{1000}}$$

$$\Rightarrow \frac{3000\,X_3^{-1} + 2\times \frac{X_2}{1000}}{3000\,X_3^{-1}} X_4 \le 1$$

$$\Rightarrow \boxed{X_4 + 6.667\times 10^{-7}\,X_2 X_3 X_4 \le 1}$$

Minimize

$$0.491389 X_1 X_2^{2/3} X_3^3 + 200\,X_1 X_2 + 0.732 X_1 X_2^{2/3} X_3^3 X_4$$

subject to

$$2.74\times 10^6\,X_1^{-1} X_2^{-1} X_3^{-1}\ X_4^{-1} \le 1$$

$$X_4 + 6.667\times 10^{-7}\,X_2 X_3 X_4 \le 1$$

$$X_j > 0,\ \ j = 1,2,3,4$$

 # terms = 6
 # variables = 4

Degrees of difficulty: 6 - (1+4) = 1

Cargo Ship Design

Number of variables: 4
Number of posynomials: 3
Total number of terms: 6
Degrees of difficulty: 1
Terms per posynomial: 3 1 2

Coefficients and exponent matrix:

t	p	Ct	exponents			
1	1	4.900000000E¯1	1	0.666667	3	0
2	1	2.000000000E2	1	1	0	0
3	1	7.300000000E¯1	1	0.666667	3	1
4	2	2.740000000E6	¯1	¯1	¯1	¯1
5	3	1.000000000E0	0	0	0	1
6	3	6.666700000E¯7	0	1	1	1

t = term number
p = posynomial
Ct = coefficient

Cargo Ship Design

Posynomial	Tolerance
1	1.00E¯4
2	1.00E¯4
3	1.00E¯4

Frequencies for
Discarding unused grid pts: 10
Reporting solutions: 1
Types of grid points to be generated: 0 1

Iteration 1

LP Solution

Col	Posy	Type	Value
1	0	0	17.58899047
8	1	0	.45000005
9	3	0	.19999982
5	2	0	1.00000000
6	3	0	.65000017
3	1	0	.54999995

Determinant of the basis matrix = 1.11111

Exponentiating
LP Dual solution

First iteration: approximation of the primal solution is:

i	X[i]	
1	7.30670E0	*# of ships*
2	2.97886E4	*cargo load capacity (tons)*
3	1.25886E1	*ship speed (knots)*
4	1.00000E0	*fraction of time en route*

Constraints

#	P	Infeas	Lambda
2	1.000000	6.88338E¯15	1.00000E0
3	1.250000	2.50000E¯1	8.50000E¯1

Weights of terms (ρ):

k	1	2	3
1	1.1322E¯1	7.1810E¯1	0.16868
2	1.0000E0		
3	8.0000E¯1	2.0000E¯1	

Objective functions:

Primal: 6.06204E7   Dual: 4.35313E7
Duality Gap: 1.70891E7 = 39.2571 percent

Type-1 grid points (#10 11) added for posynomials 1 3

Iteration 2

LP Solution

Col	Posy	Type	Value
1	0	0	18.01613928
8	1	0	.04703196
11	3	1	.49999955
5	2	0	1.00000000
6	3	0	.32357630
10	1	1	.95296804

Determinant of the basis matrix = 0.256508

Exponentiating
LP Dual solution

i	X[i]
1	2.22983E1
2	8.42986E3
3	1.45767E1
4	1.00000E0

Constraints

#	P	Infeas	Lambda
2	1.000000	8.88178E¯15	1.00000E0
3	1.081920	8.19200E¯2	8.23576E¯1

Weights of terms (p):

k	1	2	3
1	1.9336E¯1	5.1858E¯1	0.28806
2	1.0000E0		
3	9.2428E¯1	7.5717E¯2	

Objective functions:
  Primal: 7.2494E7   Dual: 6.67283E7
  Duality Gap: 5.76577E6 = 8.64067 percent
Type-1 grid points (#12 13) added for posynomials 1 3
CPU time: 26.483 sec.

i	X[i]
1	1.34326E1
2	1.65078E4
3	1.40259E1
4	8.80992E¯1

Constraints

k	P[i]	Infeas	Lambda
2	1.000000	0.00000E0	1.00000E0
3	1.000068	6.77617E¯5	8.29749E¯1

Weights of terms (p):

k	1	2	3
1	1.2965E¯1	7.0078E¯1	1.6957E¯1
2	1.0000E0		
3	8.7787E¯1	1.2213E¯1	

Objective functions:

Primal: 7.15131E7 Dual: 7.15101E7
Duality Gap: 3004.09 = 0.00420093 percent

*** Terminated at iteration 8 ***
Converged: Tolerances are satisfied

Primal feasible solution

	i	X[i]
# of ships	1	1.49566E1
cargo load (tons)	2	1.67546E4
ship speed (knots)	3	1.24553E1
fraction of time enroute	4	8.77868E¯1

(Sum of absolute differences between X & Xfeas is 0.0010729)

Objective function = 7.15172E7

Constraints

#	G[i]	Infeas	Lambda
2	1.00000E0	5.55112E¯15	1.00000E0
3	1.00000E0	0.00000E0	8.29749E¯1

Dual Solution: Cargo Ship

Weights of terms (ρ):

k	1	2	3
1	1.2965E¯1	7.0078E¯1	1.6957E¯1
2	1.0000E0		
3	8.7787E¯1	1.2213E¯1	

i.e., the ships' power plants will cost 12.965% of the total cost, the ships will cost 70.078% of the total cost, and the fuel will cost 16.957% of the total cost.