

Sensitivity Analysis in Geometric Programming



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Approximating the Effects of Inflation

Suppose that we have found the optimal solution of a GP problem, and we wish to test its sensitivity to an increase in a cost coefficient C .

Let

$$\begin{array}{l} \textit{known} \\ \left\{ \begin{array}{l} C_t = \text{original cost coefficient} \\ \delta^* = \text{optimal dual solution of original problem} \\ y^* = \text{optimal cost of original problem} \end{array} \right. \end{array}$$

$$\begin{array}{l} \textit{unknown} \\ \left\{ \begin{array}{l} C'_t = \text{cost coefficient after inflation} \\ y^{*'} = \text{optimal cost after inflation} \end{array} \right. \end{array}$$

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$$\begin{aligned}
 y^* &= \prod_{i=1}^N \left(\frac{C_i}{\delta_i^*} \right)^{\delta_i^*} \prod_{k=0}^P \lambda_k^{*\lambda_k^*} \\
 &= \left(\frac{C_t}{\delta_t^*} \cdot \frac{C_t'}{C_t} \right)^{\delta_t^*} \prod_{\substack{i=1 \\ i \neq t}}^N \left(\frac{C_i}{\delta_i^*} \right)^{\delta_i^*} \prod_{k=0}^P \lambda_k^{*\lambda_k^*} = \left(\frac{C_t'}{C_t} \cdot \frac{C_t}{\delta_t^*} \right)^{\delta_t^*} \prod_{\substack{i=1 \\ i \neq t}}^N \left(\frac{C_i}{\delta_i^*} \right)^{\delta_i^*} \prod_{k=0}^P \lambda_k^{*\lambda_k^*} \\
 &= \underbrace{\left(\frac{C_t}{C_t'} \right)^{\delta_t^*} \left(\frac{C_t'}{\delta_t^*} \right)^{\delta_t^*} \prod_{\substack{i=1 \\ i \neq t}}^N \left(\frac{C_i}{\delta_i^*} \right)^{\delta_i^*} \prod_{k=0}^P \lambda_k^{*\lambda_k^*}}_{\text{Dual objective value of new problem, evaluated at the original dual solution}} \leq \left(\frac{C_t}{C_t'} \right)^{\delta_t^*} y^{*'}
 \end{aligned}$$

Dual objective value of new problem, evaluated at the original dual solution

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$$y^* \leq \left(\frac{C_t}{C_t'} \right)^{\delta_t^*} y^{*'} \Rightarrow y^{*'} \geq \left(\frac{C_t'}{C_t} \right)^{\delta_t^*} y^*$$

That is, the minimum cost of the problem has increased by *at least* a factor of $\left(\frac{C_t'}{C_t} \right)^{\delta_t^*}$

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Example Cargo Ship Design Problem

The optimal cost is \$71517156

The optimal dual variables for terms in the objective function are

$$\delta_1^* = 0.129647 \quad \textit{cost of power plants}$$

$$\delta_2^* = 0.700782 \quad \textit{cost of ships}$$

$$\delta_3^* = 0.16957 \quad \textit{cost of fuel}$$

Estimate the optimal cost if the cost of fuel were to increase by 20%.

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The optimal primal variables are

14.9566	<i>number of ships</i>
16754.6	<i>cargo load capacity (tons)</i>
12.4553	<i>speed (knots)</i>
0.877868	<i>fraction of time en route</i>

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Lower Bound

$$y^{*'} \geq \left(\frac{C'_t}{C_t} \right)^{\delta_t^*} y^*$$

$$\begin{aligned} y^{*'} &\geq (1.2)^{0.16957} \times 71517156 \\ &= 1.0314 \times 71517156 = 73762736 \end{aligned}$$

i.e., the cost will increase by at least 3.14% .

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Upper Bound

Since the previous solution is still feasible, it provides us with an upper bound on the new cost: 73982323

That is, the cost should not increase more than 3.447%

$$\text{(since } \frac{73982323}{71517156} = 1.03447 \text{)}$$

Thus, we are able to estimate that

$$73762736 \leq y^{*'} \leq 73982323$$

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If we solve the GP with the new cost coefficient for fuel, we obtain the minimum cost 73897491 an increase of 3.328%.

The optimal primal variables are now

14.9556	<i>number of ships</i>
17383.5	<i>cargo load capacity (tons)</i>
12.0047	<i>speed (knots)</i>
0.877928	<i>fraction of time en route</i>