Dantzig-Wolfe Decomposition: an Example

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Depot Maintenance Planning

A government agency has a fleet of vehicles of three types, and three maintenance depots (denoted by A, B, & C).

Vehicle Type	Repairable Inventory	Value
1	466	3.1 $K
2	1782	8.4 $K
3	282	5.7 $K

The agency wishes to assign vehicles to the depots for repair so as to minimize the value of the nonrepaired vehicles.

Repairs require varying amounts of resources at the depots:

Resource	Vehicle 1	2	3
A1	92	—	—
A2	38	—	28
A3	—	498	—
A4	—	347	26

Resource	Vehicle 1	2	3
B1	180	5	—
B2	—	256	150
B3	—	45	80

Resource	Vehicle 2	3
C1	30	50
C2	20	20
C3	60	—

Depot C is unable to repair vehicle #1

Decision Variables

X_{1A} = # of type 1 vehicles assigned to depot A

$\vdots$

X_{3C} = # of type 3 vehicles assigned to depot C

S_1 = # of type 1 vehicles unassigned
S_2 = # of type 2 vehicles unassigned
S_3 = # of type 3 vehicles unassigned

"slack variables"

Constraints

$$\left.\begin{aligned} X_{1A} + X_{1B} + S_1 &= 466 \\ X_{2A} + X_{2B} + X_{2C} + S_2 &= 1782 \\ X_{3A} + X_{3B} + X_{3C} + S_3 &= 282 \end{aligned}\right\}$$

Required Maintenance

$$\left.\begin{aligned} 92\,X_{1A} &\le 17600 \\ 38\,X_{1A} + 28\,X_{3A} &\le 28100 \\ 498\,X_{2A} &\le 29900 \\ 347\,X_{2A} + 26\,X_{3A} &\le 30200 \end{aligned}\right\}$$

Resources at Depot A

etc.

Depot Maintenance Planning

Your problem, before adding slack &/or surplus variables:

0	0	0	0	0	0	0	0	3.1	8.4	5.7	(min)
1	0	0	1	0	0	0	0	1	0	0	= 466
0	1	0	0	1	0	1	0	0	1	0	= 1782
0	0	1	0	0	1	0	1	0	0	1	= 282
92	0	0	0	0	0	0	0	0	0	0	≤ 17600
38	0	28	0	0	0	0	0	0	0	0	≤ 28100
0	498	0	0	0	0	0	0	0	0	0	≤ 29900
0	347	26	0	0	0	0	0	0	0	0	≤ 30200
0	0	0	180	5	0	0	0	0	0	0	≤ 31600
0	0	0	0	256	150	0	0	0	0	0	≤ 41250
0	0	0	0	45	80	0	0	0	0	0	≤ 17600
0	0	0	0	0	0	30	50	0	0	0	≤ 11250
0	0	0	0	0	0	20	20	0	0	0	≤ 24600
0	0	0	0	0	0	60	0	0	0	0	≤ 40500

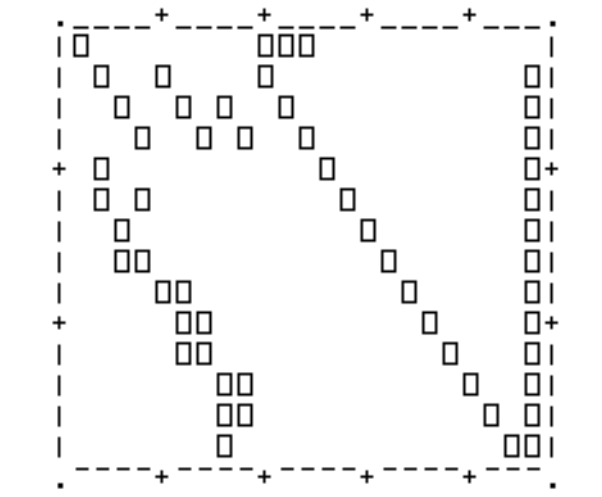
>>> TABLEAU <<<

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	B
1	0	0	0	0	0	0	0	0	3.1	8.4	5.7	0	0	0	0	0	0	0	0	0	0	0
0	1	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	466
0	0	1	0	0	1	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1780
0	0	0	1	0	0	1	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	282
0	92	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	17600
0	38	0	28	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	28100
0	0	498	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	29900
0	0	347	26	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	30200
0	0	0	0	180	5	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	31600
0	0	0	0	0	256	150	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	41300
0	0	0	0	0	45	80	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	17600
0	0	0	0	0	0	0	30	50	0	0	0	0	0	0	0	0	0	0	1	0	0	11300
0	0	0	0	0	0	0	20	20	0	0	0	0	0	0	0	0	0	0	0	1	0	24600
0	0	0	0	0	0	0	60	0	0	0	0	0	0	0	0	0	0	0	0	0	1	40500

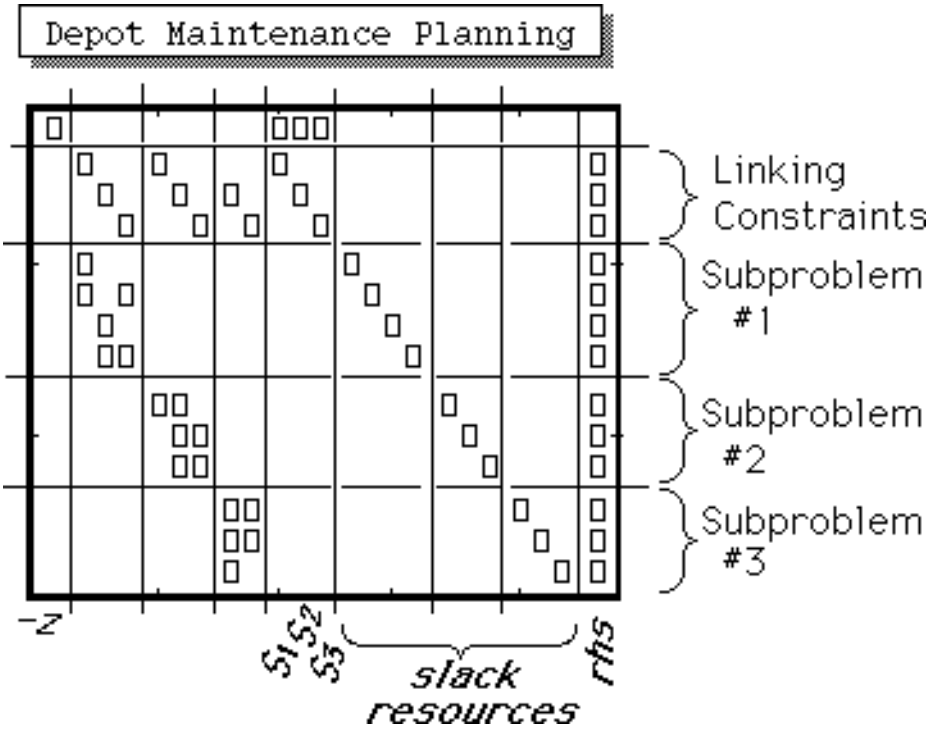
-z

slack resources

Depot Maintenance Planning



□ represents a nonzero value



Specification of Decomposition

```
Subproblem number 1:
Rows 5 6 7 8
Variables 2 3 4 13 14 15 16

Subproblem number 2:
Rows 9 10 11
Variables 5 6 7 17 18 19

Subproblem number 3:
Rows 12 13 14
Variables 8 9 20 21 22

The master problem includes rows 1 2 3 4
 and variables 1 10 11 12
(Variable #1 is -Z, the objective)
```

Master Problem Tableau

Infeasible!

No proposals from subproblems have been added yet!

1	2	3	4	B
1	3.1	8.4	5.7	0
0	1	0	0	466
0	0	1	0	1782
0	0	0	1	282
0	0	0	0	1
0	0	0	0	1
0	0	0	0	1

Let's begin by adding from each subproblem (depot) the proposal to "do nothing"

Current List of Subproblem Proposals

From Subproblem Number 1

No.	Proposal (X)						
	2	3	4	13	14	15	16
1	0	0	0	17600	28100	29900	30200

From Subproblem Number 2

No.	Proposal (X)					
	5	6	7	17	18	19
2	0	0	0	31600	41250	17600

Only the slack variables at depots are nonzero.

From Subproblem Number 3

No.	Proposal (X)				
	8	9	20	21	22
3	0	0	11250	24600	40500

Master Problem Tableau

1	2	3	4	5	6	7	B
1	3.1	8.4	5.7	0	0	0	0
0	1	0	0	0	0	0	466
0	0	1	0	0	0	0	1782
0	0	0	1	0	0	0	282
0	0	0	0	1	0	0	1
0	0	0	0	0	1	0	1
0	0	0	0	0	0	1	1

convexity constraints

initial proposals

Dantzig-Wolfe Master Problem Solution

Objective Function = 18020.8

Master Variable Values Are X[1 10 11 12] =

-18020.8 466 1782 282

Proposal Weights

s	p	wt
1	1	1.000000
2	2	1.000000
3	3	1.000000

(only one feasible solution, which is therefore optimal!)

s=Subproblem no.
p=Proposal no.
wt=Weight

Weighted Sum of the Proposals

```
---Master Variables: X[1 10 11 12] =
-18020.8 466 1782 282

---Subproblem 1 Variables: X[2 3 4 13 14 15 16] =
0 0 0 17600 28100 29900 30200

---Subproblem 2 Variables: X[5 6 7 17 18 19] =
0 0 0 31600 41250 17600

---Subproblem 3 Variables: X[8 9 20 21 22] =
0 0 11250 24600 40500

Shadow Prices of Shared Resources = 3.1 8.4 5.7      ← ω
and Simplex Multipliers of Convexity Rows are 0 0 0  ← α
```

$$\pi = [\omega, \alpha]$$

Subproblem 1

```
Defined by: Rows 5 6 7 8
     & Columns 2 3 4 13 14 15 16

Cost vector is 0 0 0 0 0 0 0
Simplex multipliers are
     3.1 8.4 5.7 for linking constraints
     0 for the convexity (GUB) constraint
Cost of shared resources per subproblem variable:
     3.1 8.4 5.7 0 0 0 0

The resulting subproblem objective function is
     -3.1 -8.4 -5.7 0 0 0 0

Optimal proposal from this subproblem is
    X[2 3 4 13 14 15 16]=
      0 11.8361 1003.57 17600 0 24005.6 0
Its cost is 0
Its reduced cost is -5819.78
```

Addition of Proposal to Master Problem

```
Proposal is X[2 3 4 13 14 15 16] =
     0 11.8361 1003.57 17600 0 24005.6 0
from subproblem # 1
```

i.e. Depot A proposes to repair
no vehicles of type 1,
11.8361 of type 2,
1003.57 of type 3

```
Its actual cost is 0
Its shared resource usages are  0 11.8361 1003.57

This is proposal number 4
Cost of shared resources: 5819.78
Reduced cost: ¯5819.78
```


Subproblem 2

```
Defined by: Rows 9 10 11
     & Columns 5 6 7 17 18 19
Cost vector is 0 0 0 0 0 0
Simplex multipliers are
     3.1 8.4 5.7 for linking constraints
     0 for the convexity (GUB) constraint
Cost of shared resources per subproblem variable:
     3.1 8.4 5.7 0 0 0

The resulting subproblem objective function is
     ¯3.1 ¯8.4 ¯5.7 0 0 0

Optimal proposal from this subproblem is X[5 6 7 17 18 19] =
     174.22 48.0699 192.961 0 0 0
Its cost is 0
Its reduced cost is ¯2043.75
```

Addition of Proposal to Master Problem

```
Proposal is X[5 6 7 17 18 19] =
     174.22 48.0699 192.961 0 0 0
from subproblem # 2
```

Depot B proposes to repair
174.22 vehicles of type 1
48.06 vehicles of type 2
192.96 vehicles of type 3

```
Its actual cost is 0
Its shared resource usages are
174.22 48.0699 192.961
This is proposal number 5
Cost of shared resources: 2043.75
Reduced cost: ¯2043.75
```


Subproblem 3

```
Defined by: Rows 12 13 14
     & Columns 8 9 20 21 22
Cost vector is 0 0 0 0 0
Simplex multipliers are
     3.1 8.4 5.7 for linking constraints
     0 for the convexity (GUB) constraint
Cost of shared resources per subproblem variable:
     8.4 5.7 0 0 0

The resulting subproblem objective function is
     ¯8.4 ¯5.7 0 0 0

Optimal proposal from this subproblem is X[8 9 20 21 22] =
       375 0 0 17100 18000
Its cost is 0
Its reduced cost is ¯3150
```

Addition of Proposal to Master Problem

Proposal is X[8 9 20 21 22] = 375 0 0 17100 18000
from subproblem # 3

Depot C proposes to repair
375 vehicles of type 2
no vehicles of type 3

Its actual cost is 0
Its shared resource usages are
0 375 0
This is proposal number 6
Cost of shared resources: 3150
Reduced cost: -3150

Master Problem Tableau

1	2	3	4	5	6	7	8	9	10	B
1	3.1	8.4	5.7	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	174.22	0	466
0	0	1	0	0	0	0	11.8361	48.0699	375	1782
0	0	0	1	0	0	0	1003.57	192.961	0	282
0	0	0	0	1	0	0	1	0	0	1
0	0	0	0	0	1	0	0	1	0	1
0	0	0	0	0	0	1	0	0	1	1

new proposals

Dantzig-Wolfe Master Problem Solution

Objective Function = 12310.7

Master Variable Values Are X[1 10 11 12] =
¯12310.7 291.78 1357.88 0

Proposal Weights

s	p	wt
1	4	0.088722
1	1	0.911278
2	5	1.000000
3	6	1.000000

s=Subproblem no.
p=Proposal no.
wt=Weight

Master problem uses proposals from depots B&C "as submitted", but only 8.87% of depot A's proposal

Weighted Sum of the Proposals

---Master Variables: X[1 10 11 12] =
¯12310.7 291.78 1357.88 0

---Subproblem 1 Variables: X[2 3 4 13 14 15 16] =
0 1.05013 89.0393 17600 25606.9 29377 27520.6

---Subproblem 2 Variables: X[5 6 7 17 18 19] =
174.22 48.0699 192.961 0 0 0

---Subproblem 3 Variables: X[8 9 20 21 22] =
375 0 0 17100 18000

Shadow Prices of Shared Resources = 3.1 8.4 ¯0.0990698
and Simplex Multipliers of Convexity Rows are
0 ¯924.754 ¯3150

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Subproblem 1

```
Defined by: Rows 5 6 7 8
     & Columns 2 3 4 13 14 15 16
Cost vector is 0 0 0 0 0 0 0
Simplex multipliers are
     3.1 8.4 -0.0990698 for linking constraints
     0 for the convexity (GUB) constraint
Cost of shared resources per subproblem variable:
     3.1 8.4 -0.0990698 0 0 0 0

The resulting subproblem objective function is
     -3.1 -8.4 0.0990698 0 0 0 0

Optimal proposal from this subproblem is
     X[2 3 4 13 14 15 16]=
     191.304 60.0402 0 0 20830.4 0 9366.06

Its cost is 0
Its reduced cost is -1097.38
```


Addition of Proposal to Master Problem

```
Proposal is X[2 3 4 13 14 15 16] =
     191.304 60.0402 0 0 20830.4 0 9366.06
from subproblem # 1

Its actual cost is 0
Its shared resource usages are
     191.304 60.0402 0

This is proposal number 7

Cost of shared resources: 1097.38
Reduced cost: -1097.38
```

Subproblem 2

```
Defined by: Rows 9 10 11
     & Columns 5 6 7 17 18 19
Cost vector is 0 0 0 0 0 0
Simplex multipliers are
     3.1 8.4 -0.0990698 for linking constraints
     -924.754 for the convexity (GUB) constraint
Cost of shared resources per subproblem variable:
     3.1 8.4 -0.0990698 0 0 0

The resulting subproblem objective function is
     -3.1 -8.4 0.0990698 0 0 0

Optimal proposal from this subproblem is
     X[5 6 7 17 18 19] =
     171.08 161.133 0 0 0 10349

Its cost is 0
Its reduced cost is -959.109
```


Addition of Proposal to Master Problem

```
Proposal is X[5 6 7 17 18 19] =
     171.08 161.133 0 0 0 10349
from subproblem # 2
Its actual cost is 0
Its shared resource usages are
171.08 161.133 0

This is proposal number 8

Cost of shared resources: 1883.86
Reduced cost: -959.109
```

Subproblem 3

```
Defined by: Rows 12 13 14
     & Columns 8 9 20 21 22
Cost vector is 0 0 0 0 0
Simplex multipliers are
     3.1 8.4 ¯0.0990698 for linking constraints
     ¯3150 for the convexity (GUB) constraint
Cost of shared resources per subproblem variable:
     8.4 ¯0.0990698 0 0 0

The resulting subproblem objective function is
     ¯8.4 0.0990698 0 0 0

Optimal proposal from this subproblem is X[8 9 20 21 22] =
     375 0 0 17100 18000

Its cost is 0
Its reduced cost is ¯4.54747E¯13
```


Master Problem Tableau

1	2	3	4	5	6	7	8	9	1 0	1 1	1 2	B
1	3.1	8.4	5.7	0	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	174.22	0	191.304	171.08	466
0	0	1	0	0	0	0	11.8361	48.0699	375	60.0402	161.133	1782
0	0	0	1	0	0	0	1003.57	192.961	0	0	0	282
0	0	0	0	1	0	0	1	0	0	1	0	1
0	0	0	0	0	1	0	0	1	0	0	1	1
0	0	0	0	0	0	1	0	0	1	0	0	1

Dantzig-Wolfe Master Problem Solution

Objective Function = 10562.6
Master Variable Values Are X[1 10 11 12] =
¯10562.6 157.372 1199.37 0

Proposal Weights

s	p	wt
1	4	0.280996
1	7	0.719004
2	8	1.000000
3	6	1.000000

s=Subproblem no.
p=Proposal no.
wt=Weight

Weighted Sum of the Proposals

---Master Variables: X[1 10 11 12] =
¯10562.6 157.372 1199.37 0

---Subproblem 1 Variables: X[2 3 4 13 14 15 16] =
137.549 46.495 282 4945.54 14977.2 6745.49 6734.23

---Subproblem 2 Variables: X[5 6 7 17 18 19] =
171.08 161.133 0 0 0 10349

---Subproblem 3 Variables: X[8 9 20 21 22] =
375 0 0 17100 18000

Shadow Prices of Shared Resources = 3.1 8.4 0.994406
and Simplex Multipliers of Convexity Rows are
¯1097.38 ¯1883.86 ¯3150

Subproblem 1

```
Defined by: Rows 5 6 7 8
     & Columns 2 3 4 13 14 15 16
Cost vector is 0 0 0 0 0 0 0
Simplex multipliers are
     3.1 8.4 0.994406 for linking constraints
     -1097.38 for the convexity (GUB) constraint
Cost of shared resources per subproblem variable:
     3.1 8.4 0.994406 0 0 0 0

The resulting subproblem objective function is
     -3.1 -8.4 -0.994406 0 0 0 0

Optimal proposal from this subproblem is
     X[2 3 4 13 14 15 16]=
     191.304 31.2895 743.944 0 0 14317.8 0

Its cost is 0
Its reduced cost is -498.277
```


Subproblem 2

```
Defined by: Rows 9 10 11
     & Columns 5 6 7 17 18 19
Cost vector is 0 0 0 0 0 0
Simplex multipliers are
     3.1 8.4 0.994406 for linking constraints
     -1883.86 for the convexity (GUB) constraint
Cost of shared resources per subproblem variable:
     3.1 8.4 0.994406 0 0 0

The resulting subproblem objective function is
     -3.1 -8.4 -0.994406 0 0 0

Optimal proposal from this subproblem is X[5 6 7 17 18 19] =
     171.08 161.133 0 0 0 10349

Its cost is 0
Its reduced cost is 0
```

Subproblem 3

```
Defined by: Rows 12 13 14
     & Columns 8 9 20 21 22
Cost vector is 0 0 0 0 0
Simplex multipliers are
     3.1 8.4 0.994406 for linking constraints
     -3150 for the convexity (GUB) constraint
Cost of shared resources per subproblem variable:
     8.4 0.994406 0 0 0

The resulting subproblem objective function is
     -8.4 -0.994406 0 0 0

Optimal proposal from this subproblem is X[8 9 20 21 22] =
     375 0 0 17100 18000

Its cost is 0
Its reduced cost is -4.54747E-13
```

(= zero)

Master Problem Tableau

1	2	3	4	5	6	7	8	9	10	11	12	13	B
1	3.1	8.4	5.7	0	0	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	174.22	0	191.30	171.08	191.30	466
0	0	1	0	0	0	0	11.83	48.06	375	60.04	161.13	31.28	1782
0	0	0	1	0	0	0	1003.57	192.96	0	0	0	743.94	282
0	0	0	0	1	0	0	1	0	0	1	0	1	1
0	0	0	0	0	1	0	0	1	0	0	1	0	1
0	0	0	0	0	0	1	0	0	1	0	0	0	1

Dantzig-Wolfe Master Problem Solution

Objective Function = 10373.7
Master Variable Values Are X[1 10 11 12] =
¯10373.7 103.616 1196.73 0

Proposal Weights

s	p	wt
1	9	0.379061
1	7	0.620939
2	8	1.000000
3	6	1.000000

s=Subproblem no.
p=Proposal no.
wt=Weight

Weighted Sum of the Proposals

---Master Variables: X[1 10 11 12] =
¯10373.7 103.616 1196.73 0

---Subproblem 1 Variables: X[2 3 4 13 14 15 16] =
191.304 49.1419 282 0 12934.4 5427.33 5815.76

---Subproblem 2 Variables: X[5 6 7 17 18 19] =
171.08 161.133 0 0 0 10349

---Subproblem 3 Variables: X[8 9 20 21 22] =
375 0 0 17100 18000

Shadow Prices of Shared Resources = 3.1 8.4 0.324629
and Simplex Multipliers of Convexity Rows are
¯1097.38 ¯1883.86 ¯3150

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Subproblem 1

```
Defined by: Rows 5 6 7 8
     & Columns 2 3 4 13 14 15 16
Cost vector is 0 0 0 0 0 0 0
Simplex multipliers are
     3.1 8.4 0.324629 for linking constraints
     -1097.38 for the convexity (GUB) constraint
Cost of shared resources per subproblem variable:
     3.1 8.4 0.324629 0 0 0 0

The resulting subproblem objective function is
     -3.1 -8.4 -0.324629 0 0 0 0

Optimal proposal from this subproblem is
     X[2 3 4 13 14 15 16]=
     191.304 60.0402 360.233 0 10743.9 0 0

Its cost is 0
Its reduced cost is -116.942
```


Subproblem 2

```
Defined by: Rows 9 10 11
     & Columns 5 6 7 17 18 19
Cost vector is 0 0 0 0 0 0
Simplex multipliers are
     3.1 8.4 0.324629 for linking constraints
     -1883.86 for the convexity (GUB) constraint
Cost of shared resources per subproblem variable:
     3.1 8.4 0.324629 0 0 0

The resulting subproblem objective function is
     -3.1 -8.4 -0.324629 0 0 0

Optimal proposal from this subproblem is
     X[5 6 7 17 18 19] =
     171.08 161.133 0 0 0 10349

Its cost is 0
Its reduced cost is 0
```

Subproblem 3

```
Defined by: Rows 12 13 14
     & Columns 8 9 20 21 22
Cost vector is 0 0 0 0 0
Simplex multipliers are
     3.1 8.4 0.324629 for linking constraints
     -3150 for the convexity (GUB) constraint
Cost of shared resources per subproblem variable:
     8.4 0.324629 0 0 0

The resulting subproblem objective function is
     -8.4 -0.324629 0 0 0


Optimal proposal from this subproblem is
     X[8 9 20 21 22] =
     375 0 0 17100 18000
Its cost is 0
Its reduced cost is -4.54747E-13
```


1	2	3	4	5	6	7	8	9	1 0	1 1	1 2
1	3.1	8.4	5.7	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	174.22	0	191.30	171.08
0	0	1	0	0	0	0	11.83	48.06	375	60.04	161.13
0	0	0	1	0	0	0	1003.57	192.96	0	0	0
0	0	0	0	1	0	0	1	0	0	1	0
0	0	0	0	0	1	0	0	1	0	0	1
0	0	0	0	0	0	1	0	0	1	0	0

Master Problem Tableau

1 3	1 4	B
0	0	0
191.30	191.30	466
31.28	60.04	1782
743.94	360.23	282
1	1	1
0	0	1
0	0	1

Dantzig-Wolfe Master Problem Solution

Objective Function = 10282.2
Master Variable Values Are X[1 10 11 12] =
¯10282.2 103.616 1185.83 0

Proposal Weights

s	p	wt
1	10	0.782826
1	7	0.217174
2	8	1.000000
3	6	1.000000

s=Subproblem no.
p=Proposal no.
wt=Weight

Weighted Sum of the Proposals

---Master Variables: X[1 10 11 12] =
¯10282.2 103.616 1185.83 0

---Subproblem 1 Variables: X[2 3 4 13 14 15 16] =
191.304 60.0402 282 0 12934.4 0 2034.06

---Subproblem 2 Variables: X[5 6 7 17 18 19] =
171.08 161.133 0 0 0 10349

---Subproblem 3 Variables: X[8 9 20 21 22] =
375 0 0 17100 18000

Shadow Prices of Shared Resources = 3.1 8.4 0
and Simplex Multipliers of Convexity Rows are
¯1097.38 ¯1883.86 ¯3150

Subproblem 1

```
Defined by: Rows 5 6 7 8
     & Columns 2 3 4 13 14 15 16
Cost vector is 0 0 0 0 0 0 0
Simplex multipliers are
     3.1 8.4 0 for linking constraints
     -1097.38 for the convexity (GUB) constraint
Cost of shared resources per subproblem variable:
     3.1 8.4 0 0 0 0 0

The resulting subproblem objective function is
     -3.1 -8.4 0 0 0 0 0

Optimal proposal from this subproblem is
     X[2 3 4 13 14 15 16]=
     191.304 60.0402 360.233 0 10743.9 0 0

Its cost is 0
Its reduced cost is 0
```


Subproblem 2

```
Defined by: Rows 9 10 11
     & Columns 5 6 7 17 18 19
Cost vector is 0 0 0 0 0 0
Simplex multipliers are
     3.1 8.4 0 for linking constraints
     -1883.86 for the convexity (GUB) constraint
Cost of shared resources per subproblem variable:
     3.1 8.4 0 0 0 0

The resulting subproblem objective function is
     -3.1 -8.4 0 0 0 0

Optimal proposal from this subproblem is
     X[5 6 7 17 18 19] =
     171.08 161.133 0 0 0 10349

Its cost is 0
Its reduced cost is 0
```

Subproblem 3

```
Defined by: Rows 12 13 14
     & Columns 8 9 20 21 22
Cost vector is 0 0 0 0 0
Simplex multipliers are
     3.1 8.4 0 for linking constraints
     -3150 for the convexity (GUB) constraint
Cost of shared resources per subproblem variable:
     8.4 0 0 0 0

The resulting subproblem objective function is
     -8.4 0 0 0 0

Optimal proposal from this subproblem is
     X[8 9 20 21 22] =
     375 0 0 17100 18000
Its cost is 0
Its reduced cost is -4.54747E-13
```


All three depots report that they are unable to find a proposal with a negative reduced cost....

Hence the latest solution to the master problem cannot be improved, and is optimal!

```
Master Variables:
X[1 10 11 12] =
-10282.2 103.616 1185.83 0
```

$S_1 = 103.6$, $S_2 = 1185.8$, $S_3 = 0$

```
Subproblem 1 Variables:
X[2 3 4 13 14 15 16] =
191.304 60.0402 282 0 12934.4 0 2034.06
```

$X_{1A} = 191.3$, $X_{2A} = 60$, $X_{3A} = 282$

```
Subproblem 2 Variables:
X[5 6 7 17 18 19] =
171.08 161.133 0 0 0 10349
```

$X_{1B} = 171.1$, $X_{2B} = 161.1$, $X_{3B} = 0$

```
Subproblem 3 Variables:
X[8 9 20 21 22] =
375 0 0 17100 18000
```

$X_{2C} = 375$, $X_{3C} = 0$

Current List of Subproblem Proposals

From Subproblem Number 1

No.	Proposal (X)						
	2	3	4	13	14	15	16
1	0	0	0	17600	28100	29900	30200
4	0	11.8361	1003.57	17600	0	24005.6	0
7	191.304	60.0402	0	0	20830.4	0	9366.06
9	191.304	31.2895	743.944	0	0	14317.8	0
10	191.304	60.0402	360.233	0	10743.9	0	0

From Subproblem Number 2

No.	Proposal (X)					
	5	6	7	17	18	19
2	0	0	0	31600	41250	17600
5	174.22	48.0699	192.961	0	0	0
8	171.08	161.133	0	0	0	10349

From Subproblem Number 3

No.	Proposal (X)				
	8	9	20	21	22
3	0	0	11250	24600	40500
6	375	0	0	17100	18000