

PRODUCTION PLANNING WITH RANDOM DEMAND & BACKORDERING



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At the beginning of each period, the quantity to be produced of an item is to be decided.

The quantity demanded in each period is random, and independent of other period's demand.

In case of a shortage, demand may be backordered, but must be filled during the next period.

Planning Horizon	=	4 periods
Maximum inventory	=	6
Maximum backorder level	=	3
Production Capacity	=	3

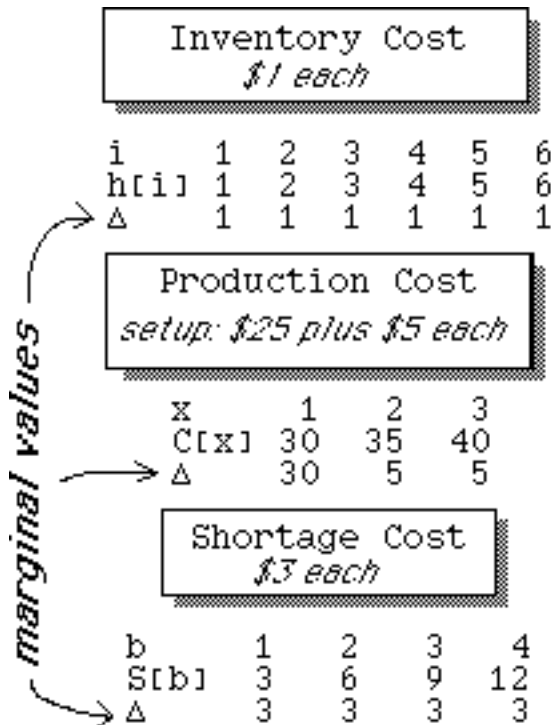
DP Model

Stage $n =$ # of periods remaining in planning period ($n=4, 3, 2, 1, 0$)

State $S_n =$ inventory position
 (positive if stock on hand,
 negative if there are backorders)
 $S_n \in \{-3, -2, -1, 0, 1, 2, 3, 4, 5, 6\}$

Decision $X_n =$ quantity to be produced

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Data

Salvage values
end of period 4

i	1	2	3	4	5	6
S(i)	3	6	9	12	15	18
Δ	3	3	3	3	3	3

Demand Distribution

d	P{d}
0	0.1 <i>independent</i>
1	0.2 <i>identically-</i>
2	0.3 <i>distributed</i>
3	0.3
4	0.1

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Optimal Value Function

$f_n(S_n)$ = minimum expected cost for the next
 n periods, given current inventory
 position is S_n
 ($S_n > 0 \implies$ stock on hand,
 $S_n < 0 \implies$ backorder position)

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Recursive Definition

$$f_n(S_n) = \begin{cases} \text{minimum}_{(-S_n)^+ \leq X_n \leq 3} \sum_{d=0}^4 p_d \{ h(S_n) + c(X_n) + s(d - S_n - X_n)^+ \\ \quad + f_{n-1}(S_n + X_n - d) \} & \text{if } n > 1 \\ -v(S_0) & \text{if } n = 0 \end{cases}$$

backorders must
 be filled in next
 period $\implies (-S_n)^+ \leq X_n \leq 3$

$h(S)$ = storage cost
 $c(X)$ = production cost
 $s(S)$ = shortage cost
 $v(S)$ = salvage value

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APL code

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▽VALUE←F_prodn N;t;C;stock_cost
  ⍝
  ⍝      Optimal Value Function of DP model
  ⍝      of the Stochastic Production Planning Problem
  ⍝      (with Backordering)
  →LAST IF N=0

  ⍝      Compute Monthly Cost
  stock_cost←SHORTCOST[1+0[-s]+G[1+0[s] ⍝ cost of stock
  C←((stock_cost°.+H)+BIG×0>s°.+x) °.+0×d

  ⍝      Compute Transition Matrix
  t←TRANSITION (1/s)[(1/s)l s °.+ x °.- d

  ⍝      Evaluate Optimal Value Function
  VALUE←P MINΔE C + (F_prodn N-1)[t]
  →0

  LAST: ⍝ Include salvage value as negative cost,
  ⍝      and include cost of production to fill
  ⍝      any remaining backorders
  VALUE←(SHORTCOST[1+0[-s]+(-SALVAGE)[1+0[s]+H[1+0[(-s)]

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Salvage value = \$3 each
 Setup cost = \$25
 unit prod'n cost = \$5

d	0	1	2	3	4
P_d	.1	.2	.3	.3	.1

Stage 1
(final period)

		x:			
	s	0	1	2	3
-3		999999.99	999999.99	999999.99	87.50
-2		999999.99	999999.99	79.50	72.80
-1		999999.99	71.50	64.80	55.80
0		38.50	56.80	47.80	40.30
1		27.80	43.80	36.30	35.30
2		14.80	32.30	31.30	33.30
3		3.30	27.30	29.30	31.30
4		-1.70	25.30	27.30	29.60
5		-3.70	23.30	25.60	28.50
6		-5.70	21.60	24.50	28.30

Example: Let's compute the expected cost for $S=1, X=2$

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Example computation: $S=1, X=2$

d	p{d}	Cost				Total	p{d}x Cost
		storage	prod'n	shortage	future		
0	0.1	1	35	0	-9	27	2.7
1	0.2	1	35	0	-6	30	6.0
2	0.3	1	35	0	-3	33	9.9
3	0.3	1	35	0	0	36	10.8
4	0.1	1	35	0	33	69	6.9
							36.3

salvage value

shortage + prod'n costs

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s	x: 0	1	2	3
-3	999999.99	999999.99	999999.99	87.50
-2	999999.99	999999.99	79.50	72.80
-1	999999.99	71.50	64.80	55.80
0	38.50	56.80	47.80	40.30
1	27.80	43.80	36.30	35.30
2	14.80	32.30	31.30	33.30
3	3.30	27.30	29.30	31.30
4	-1.70	25.30	27.30	29.60
5	-3.70	23.30	25.60	28.50
6	-5.70	21.60	24.50	28.30

Stage 1

$f_1(S_1)$

$f_1(S_1)$ is found by computing the minimum in each row

State	Optimal Values	Optimal Decisions
-3	87.50	3
-2	72.80	3
-1	55.80	3
0	38.50	0
1	27.80	0
2	14.80	0
3	3.30	0
4	-1.70	0
5	-3.70	0
6	-5.70	0

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Stage 2 Using $f_1(S_1)$, compute $f_2(S_2)$

S_1	$f_1(S_1)$
-3	87.50
-2	72.80
-1	55.80
0	38.50
1	27.80
2	14.80
3	3.30
4	-1.70
5	-3.70
6	-5.70

$x:$

S	0	1	2	3
-3	999999.99	999999.99	999999.99	120.85
-2	999999.99	999999.99	112.85	103.81
-1	999999.99	104.85	95.81	85.61
0	71.85	87.81	77.61	68.76
1	58.81	73.61	64.76	58.12
2	44.61	60.76	54.12	49.50
3	31.76	50.12	45.50	43.65
4	21.12	41.50	39.65	41.00
5	12.50	35.65	37.00	40.30
6	6.65	33.00	36.30	40.50

Example: Let's compute value for $S=-2, X=3$

Example computation: $S=-2, X=3$

d	p{d}	Cost				Total	p{d}x Cost
		storage	prod'n	shortage	future		
0	0.1	0	40	6	27.8	73.8	7.38
1	0.2	0	40	6	38.5	84.5	16.9
2	0.3	0	40	6	55.8	101.8	30.54
3	0.3	0	40	6	72.8	118.8	35.64
4	0.1	0	40	6	87.5	133.5	13.35
							103.81

S \ X:		0	1	2	3	Stage 2
		0	1	2	3	
-3		999999.99	999999.99	999999.99	120.85	$f_2(S_2)$
-2		999999.99	999999.99	112.85	103.81	
-1		999999.99	104.85	95.81	85.61	
0		71.85	87.81	77.61	68.76	
1		58.81	73.61	64.76	58.12	
2		44.61	60.76	54.12	49.50	
3		31.76	50.12	45.50	43.65	
4		21.12	41.50	39.65	41.00	
5		12.50	35.65	37.00	40.30	
6		6.65	33.00	36.30	40.50	

State	Optimal Values	Optimal Decisions
-3	120.85	3
-2	103.81	3
-1	85.61	3
0	68.76	3
1	58.12	3
2	44.61	0
3	31.76	0
4	21.12	0
5	12.50	0
6	6.65	0

Compute $f_2(S_2)$ by finding the minimum value in each row

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Stage 3

Use $f_2(S_2)$ to compute $f_3(S_3)$

S_2	$f_2(S_2)$
-3	120.85
-2	103.81
-1	85.61
0	68.76
1	58.12
2	44.61
3	31.76
4	21.12
5	12.50
6	6.65

S \ X:		0	1	2	3
		0	1	2	3
-3		999999.99	999999.99	999999.99	152.48
-2		999999.99	999999.99	144.48	134.47
-1		999999.99	136.48	126.47	115.78
0		103.48	118.48	107.78	98.72
1		89.48	103.78	94.72	87.16
2		74.78	90.72	83.16	76.20
3		61.72	79.16	72.20	66.49
4		50.16	68.20	62.49	59.26
5		39.20	58.49	55.26	54.85
6		29.49	51.26	50.85	53.23

s \ x:	0	1	2	3
-3	999999.99	999999.99	999999.99	152.48
-2	999999.99	999999.99	144.48	134.47
-1	999999.99	136.48	126.47	115.78
0	103.48	118.48	107.78	98.72
1	89.48	103.78	94.72	87.16
2	74.78	90.72	83.16	76.20
3	61.72	79.16	72.20	66.49
4	50.16	68.20	62.49	59.26
5	39.20	58.49	55.26	54.85
6	29.49	51.26	50.85	53.23

State	Optimal Values	Optimal Decisions
-3	152.48	3
-2	134.47	3
-1	115.78	3
0	98.72	3
1	87.16	3
2	74.78	0
3	61.72	0
4	50.16	0
5	39.20	0
6	29.49	0

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Stage 4

s \ x:	0	1	2	3
-3	999999.99	999999.99	999999.99	183.36
-2	999999.99	999999.99	175.36	164.78
-1	999999.99	167.36	156.78	145.71
0	134.36	148.78	137.71	128.47
1	119.78	133.71	124.47	116.81
2	104.71	120.47	112.81	105.62
3	91.47	108.81	101.62	94.83
4	79.81	97.62	90.83	85.83
5	68.62	86.83	81.83	79.47
6	57.83	77.83	75.47	76.46

S_3	$f_3(S_3)$
-3	152.48
-2	134.47
-1	115.78
0	98.72
1	87.16
2	74.78
3	61.72
4	50.16
5	39.20
6	29.49

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S \ X:	0	1	2	3
-3	999999.99	999999.99	999999.99	183.36
-2	999999.99	999999.99	175.36	164.78
-1	999999.99	167.36	156.78	145.71
0	134.36	148.78	137.71	128.47
1	119.78	133.71	124.47	116.81
2	104.71	120.47	112.81	105.62
3	91.47	108.81	101.62	94.83
4	79.81	97.62	90.83	85.83
5	68.62	86.83	81.83	79.47
6	57.83	77.83	75.47	76.46

Stage 4

$f_4(S_4)$

State	Optimal Values	Optimal Decisions
-3	220.96	3
-2	208.98	3
-1	197.60	3
0	188.38	3
1	184.38	2
2	180.38	1
3	176.38	0
4	172.93	0
5	170.18	0
6	167.80	0

Optimal Returns and Decisions

Stage 4

State	Optimal Values	Optimal Decisions
-3	220.96	3
-2	208.98	3
-1	197.60	3
0	188.38	3
1	184.38	2
2	180.38	1
3	176.38	0
4	172.93	0
5	170.18	0
6	167.80	0

Stage 3

State	Optimal Values	Optimal Decisions
-3	152.48	3
-2	134.47	3
-1	115.78	3
0	98.72	3
1	87.16	3
2	74.78	0
3	61.72	0
4	50.16	0
5	39.20	0
6	29.49	0

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Stage 2

State	Optimal Values	Optimal Decisions
-3	120.85	3
-2	103.81	3
-1	85.61	3
0	68.76	3
1	58.12	3
2	44.61	0
3	31.76	0
4	21.12	0
5	12.50	0
6	6.65	0

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Stage 1

State	Optimal Values	Optimal Decisions
-3	87.50	3
-2	72.80	3
-1	55.80	3
0	38.50	0
1	27.80	0
2	14.80	0
3	3.30	0
4	-1.70	0
5	-3.70	0
6	-5.70	0

