

Space-filling Curve Algorithm for the Traveling Salesman Problem

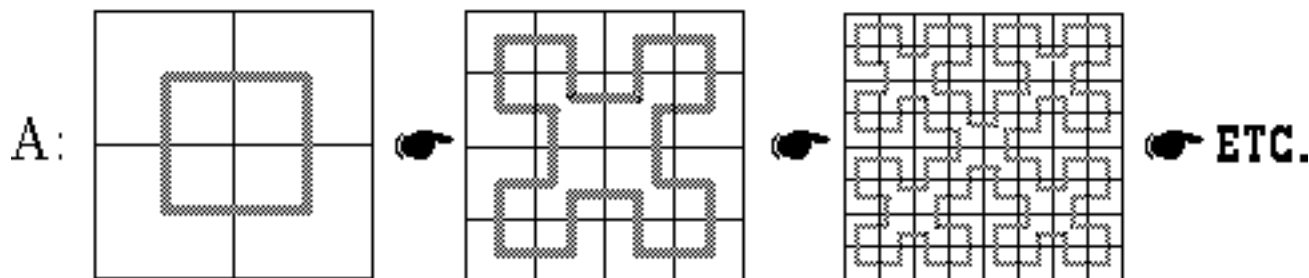


This Hypercard stack was prepared by:
Dennis L. Bricker,
Dept. of Industrial Engineering,
University of Iowa,
Iowa City, Iowa 52242
e-mail: dennis-bricker@uiowa.edu

Spacefilling Curve

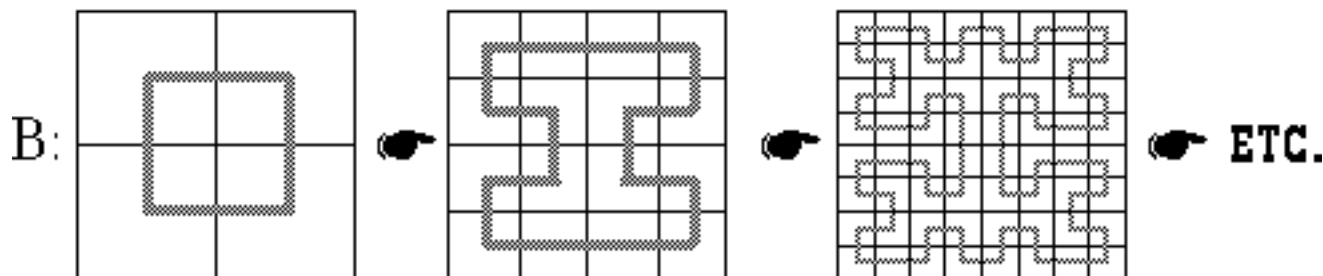
- a continuous mapping of the unit interval $[0,1]$ onto a unit hypercube (for example, a square)
- first introduced by Peano (1891), Hilbert (1891), and others as "topological monsters"
- tend to preserve "nearness" among points:
 - if 2 points are close on the curve, they are close in the square
 - if 2 points are close in the square, they are "likely" to be close on the curve.

Examples...



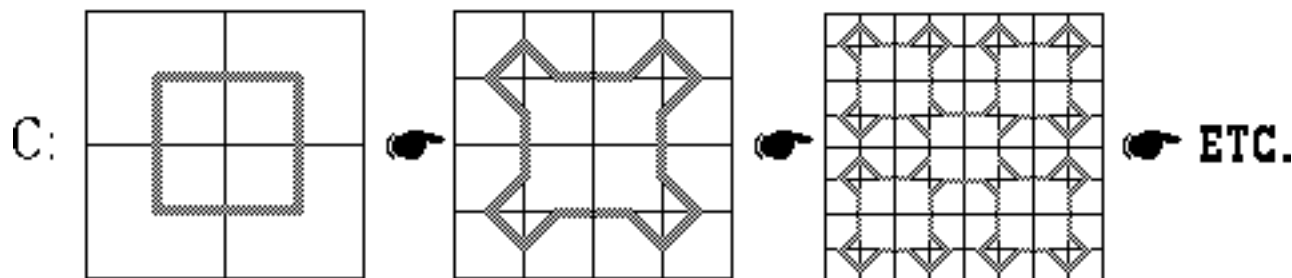
Each curve is the limit of a sequence of recursive constructions.

Examples...



Each curve is the limit of a sequence of recursive constructions.

Examples...



Each curve is the limit of a sequence of recursive constructions.

"Generic" Spacefilling Curve Heuristic:

1. Transform the problem (routing, location, etc.) in the plane, via a spacefilling curve, to a problem on the unit interval.
2. Solve the (easier) problem on the unit interval.
3. Map the solution back to the problem in the plane.

Pseudo-code to compute position on the space-filling curve (A) of a point (x,y) in the unit square:

```

      ∇z←POSITION xy;x;y;Q;T;ij
[1]  A
[2]  A          Return position of (x,y) on a
[3]  A          space-filling curve
[4]  A          (cf. Bartholdi & Platzman)
[5]  A          (Recursively-defined function)
[6]  →Compute IF eps<[ /|xy-1 1      ◇ z←0.5      ◇ →End
[7]
[8]  Compute:x←xy[1] ◇ y←xy[2]
[9]  ij←(1|12×x),(1|12×y)
[10] A Q is quadrant in which (x,y) lies
[11] Q←~1+((4 2ρ0 0 0 1 1 1 1 0)^(.=ij))∨1
[12] T←POSITION (2×|xy-0.5)
[13] →L1 IF 1≠Q|2      ◇ T←1-T
[14] L1:z←(7÷8)+(Q+T)÷4 ◇ z←z-|z
[15] End:

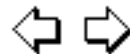
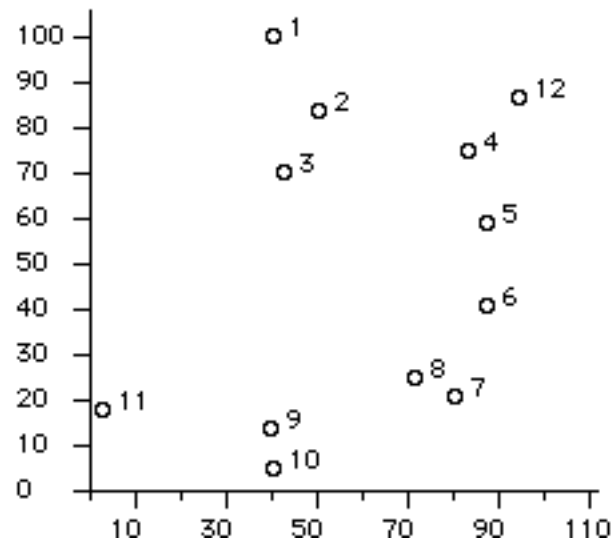
```

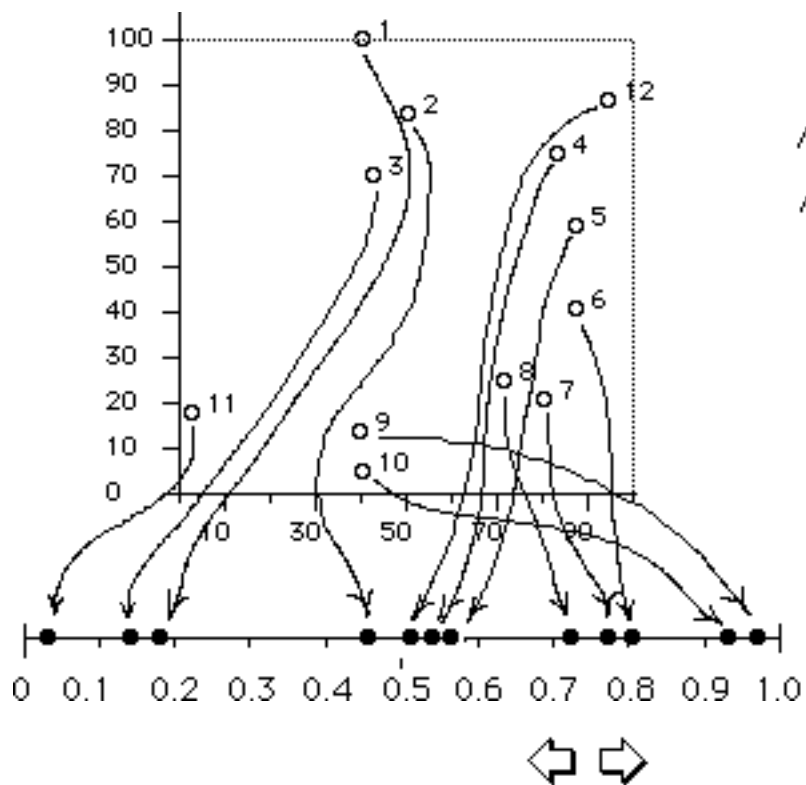
∇

APL Code

Example

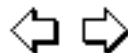
Random Symmetric TSP
(seed= 133398)





Coordinate on Space-filling Curve

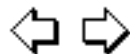
i	Xi	Yi	Position
1	41	100	0.18236921
2	51	84	0.45818231
3	43	70	0.13481542
4	84	75	0.52611921
5	88	59	0.55589939
6	88	41	0.80589939
7	81	21	0.77961296
8	72	25	0.71850459
9	40	14	0.96073832
10	41	5	0.93535641
11	3	18	0.02027783
12	95	87	0.51850270



Sort the points:

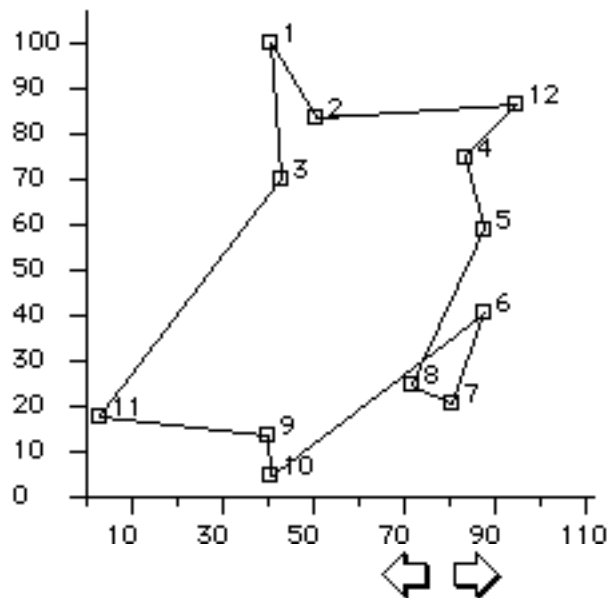
i	X_i	Y_i	Position
1	41	100	0.18236921
2	51	84	0.45818231
3	43	70	0.13481542
4	84	75	0.52611921
5	88	59	0.55589939
6	88	41	0.80589939
7	81	21	0.77961296
8	72	25	0.71850459
9	40	14	0.96073832
10	41	5	0.93535641
11	3	18	0.02027783
12	95	87	0.51850270

i	Position
11	0.02027783
3	0.13481542
1	0.18236921
2	0.45818231
12	0.51850270
4	0.52611921
5	0.55589939
8	0.71850459
7	0.77961296
6	0.80589939
10	0.93535641
9	0.96073832



This determines the order in which cities are visited...

Space-filling-curve tour: 1 2 12 4 5 8 7 6 10 9 11 3 1
with length 365



The space-filling curve has been suggested as a "low-technology" algorithm...

- 1) Find location on map and read (x,y) coordinates
- 2) Find (x,y) in table giving position
- 3) Prepare card with (x,y) and position, and insert into card file according to the position
- 4) The sorted cards give a tour.

"A Minimal Technology Routing System for Meals on Wheels",
Interfaces, June 1983 (Volume 13, no. 2), by J. Bartholdi et al.

