


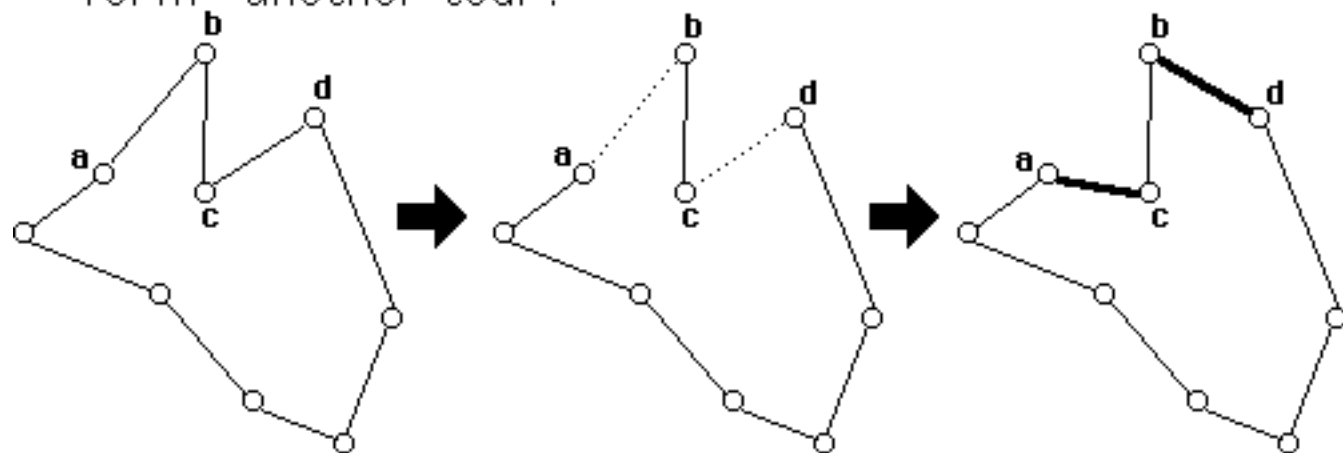
# Exchange Algorithm for the Traveling Salesman Problem



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*Exchange heuristics*, given an initial tour, try to replace  $k$  edges of the tour with  $k$  edges not on the tour in order to find a shorter tour.

A *k-exchange* is performed by deleting  $k$  edges of a tour, and reconnecting the segments so as to form another tour.



Example: a 2-exchange

Edges **ab** and **cd** are replaced by **ac** and **bd**

For a specified integer  $k$ , a  *$k$ -neighborhood* of a tour is one which might be obtained by a  $k$ -exchange.

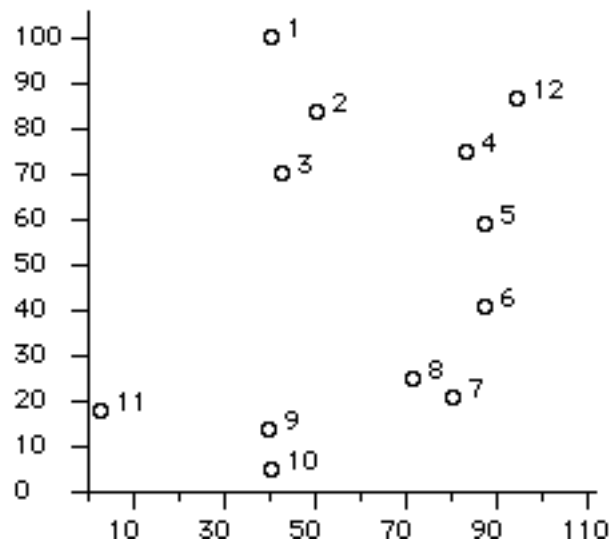
If no shorter tour exists in a  $k$ -neighborhood of a tour, that tour is said to be  *$k$ -optimal*.

(Only if a tour is  $k$ -optimal for every  $k \leq N/2$  can we be certain that the tour is truly optimal!)

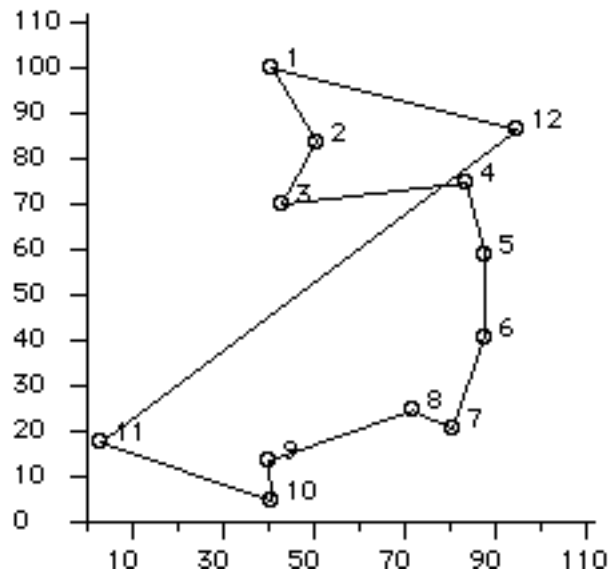


# Example

Random Symmetric TSP  
(seed= 133398)

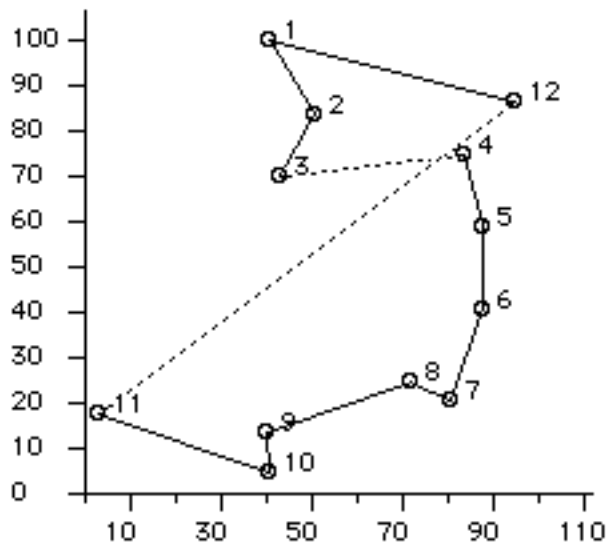


Lin's 2-exchange heuristic



*Initial tour  
(found by  
nearest  
neighbor  
heuristic)*

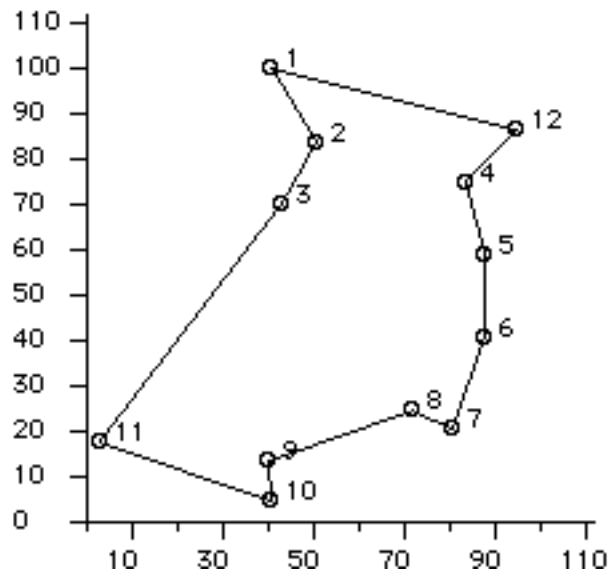
Tour # 1 is 1 2 3 / 11 10 9 8 7 6 5 4 / 12 1  
Length: 321      Improvement: 74



Edges (3,4) & (11,12) are removed, breaking tour into 2 paths. These can then be reconnected in only one other way.

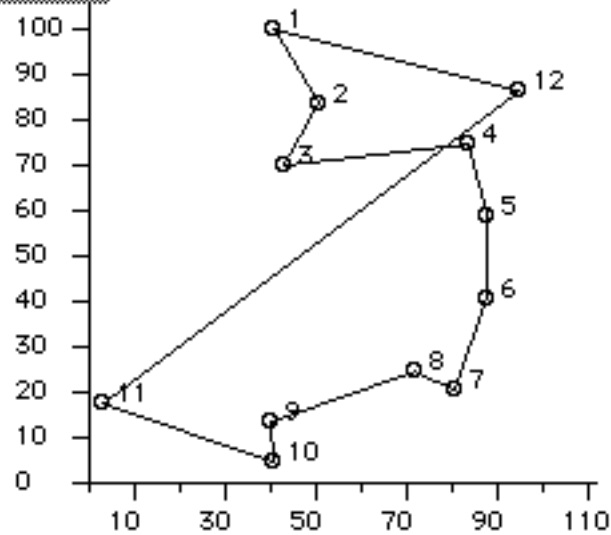


2-Optimal Tour: 1 2 3 11 10 9 8 7 6 5 4 12 1,  
with length 321



No 2-neighbor  
tour gives any  
improvement

### 3-exchange heuristic



*Initial tour  
(found by  
nearest  
neighbor  
heuristic)*

Exchange type: 1

Replace edges: 1 3 11, i.e., (1 2), (3 4), & (11 12)  
having length 175

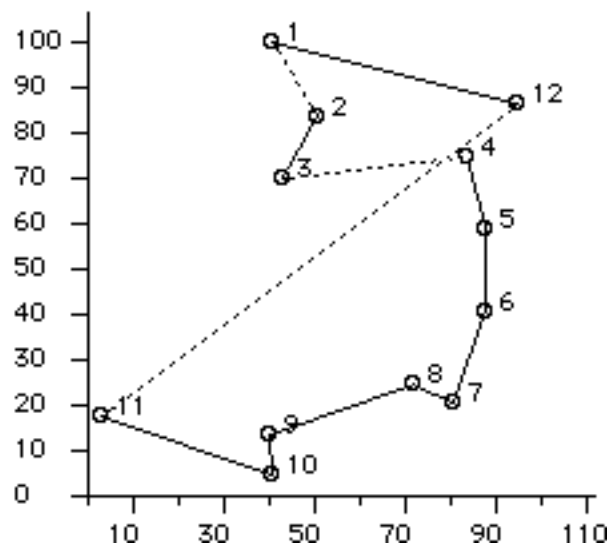
with edges (1 2), (3 11), & (4 12) having length 101

Tour # 1 is 1 2 3 11 10 9 8 7 6 5 4 12 1 with length: 321

Improvement: 74

*Because edge (1,2) was  
re-inserted, this is  
actually a 2-neighboring  
tour!*

3-exchange  
heuristic



3-Optimal Tour: 1 2 3 11 10 9 8 7 6 5 4 12 1,  
with length 321

*No further  
improvement  
was found.*

