Stochastic Transportation Problem with Recourse

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Stochastic Transportation Problem (without Simple Recourse):

Consider the following problem:

- A *single* product is stored in various quantities in a network of n nodes.
- *Before* demand for the product occurs, the product may be moved from one node to another at a known cost C^1.
- *After* demand becomes known, there is still an opportunity to move the product between nodes, but at a greater cost.
- Product at a node in excess of demand has a *salvage value*, and product sold earns a revenue.

Notation

- S_i = initial supply available at node i, $i = 1, \ldots n$
- C_{ij}^1 = *(first-stage)* transportation cost from node i to node j *before* demand is known
- C_{ij}^2 = *(second-stage)* transportation cost from node i to node j *after* demand is known
- V_i = salvage value of product at node i, $i = 1, \ldots n$
- U_i = penalty per unit shortage at node i, $i = 1, \ldots n$
- D_i = demand at node i, $i = 1, \ldots n$,
  which is *random* with discrete distribution:

$$p_i^k \equiv P\left\{D_i = d_i^k\right\},\ k = 1, \ldots K_i$$

We wish to formulate the optimization problem:

to determine the shipment plan for the product

before the demand becomes known

in order to minimize the sum of

the first-stage shipment costs

and the *expected* cost of the second-stage

(i.e., shipment costs

& shortage penalties,

minus salvage values).

Define the decision variables:

Stage 1:

X_{ij} = shipment in first stage from node i to node j
(if $i = j$, the amount retained at node i)

Stage 2:

Y_{ij} = shipment in second stage from node i to node j
Z_i^+ = quantity in excess of demand at node i after all shipments
Z_i^- = shortage at node i after all shipments

Stochastic LP model:

$$\text{Minimize} \quad \sum_{i=1}^{n}\sum_{j=1}^{n} C_{ij}^{1} X_{ij} + E_D\left\{Q\left(X,D\right)\right\}$$

$$\text{subject to} \quad \sum_{j=1}^{n} X_{ij} = S_i \quad \forall i$$

$$X_{ij} \geq 0 \quad \forall i \,\&\, j$$

where *Q(X,D)* is the minimum cost of the second stage:

$$Q\left(X,D\right) = Min \sum_{i=1}^{n}\sum_{j=1}^{n} \mathrm{C}_{\mathrm{ij}}^{2} Y_{ij} + \sum_{i=1}^{n}\left(U_i Z_i^{-} - V_i Z_i^{+}\right)$$

$$s.t. \quad \sum_{\mathrm{j=1}}^{\mathrm{n}} \mathrm{Y}_{\mathrm{ij}} - \sum_{k=1}^{n} Y_{ki} + Z_i^{-} + Z_i^{+} = D_i - \sum_{k=1}^{n} \mathrm{X}_{\mathrm{ki}} \quad \forall i$$

$$Y_{ij} \geq 0, Z_i^{+} \geq 0, Z_i^{-} \geq 0 \quad \forall i \,\&\, j$$

Example data:

$n = 3$ nodes

	Node 1	Node 2	Node 3
Supply S_i	2	4	14
Shortage penalty U_i	8	9	8
Salvage value V_i	1	4	5

Shipping costs:

Before demand occurs:

	Node 1	Node 2	Node 3
Node 1	0	2	3
Node 2	2	0	3
Node 3	3	2	0

After demand occurs:

	Node 1	Node 2	Node 3
Node 1	0	6	10
Node 2	6	0	15
Node 3	12	15	0

Demand distributions:

Node 1:

demand	4	6	8
P{demand}	0.4	0.4	0.2

Node 2:

demand	6	8
P{demand}	0.4	0.6

Node 3:

demand	6	8
P{demand}	0.5	0.5

In the notation used for the general 2-stage stochastic LP with recourse, identify the arrays

- c=
- A=
- b=
- T=
- q=
- W=
- h=

Which of these arrays are random?

Stochastic Transportation Problem, non-Simple Recourse

First-stage data:

A,B=

```
1 1 1 0 0 0 0 0 0 =  2
0 0 0 1 1 1 0 0 0 =  4
0 0 0 0 0 0 1 1 1 = 14
```

i	variable	cost
1	X11	0
2	X12	2
3	X13	3
4	X21	2
5	X22	0
6	X23	2
7	X31	3
8	X32	2
9	X33	0

Objective: Minimize

Second-stage data

K= # scenarios = 12
The following data vary by scenario: h

Costs:

i	Variable	q	
1	Y12	6	
2	Y13	10	
3	Y21	6	
4	Y23	15	
5	Y31	12	
6	Y32	15	
7	EX1	¯4	*excess*
8	EX2	¯4	*excess*
9	EX3	¯2	*excess*
10	SH1	15	*shortage*
11	SH2	20	*shortage*
12	SH3	30	*shortage*

```
Technology matrix T
(coefficients of X in 2nd stage) =
        1 0 0 1 0 0 1 0 0
        0 1 0 0 1 0 0 1 0
        0 0 1 0 0 1 0 0 1

Technology matrix W
(coefficients of Y in 2nd stage) =
        ¯1 ¯1  1  0  1  0 ¯1  0  0 1 0 0
         1  0 ¯1 ¯1  0  1  0 ¯1  0 0 1 0
         0  1  0  1 ¯1 ¯1  0  0 ¯1 0 0 1

Right-hand-sides in second stage =
```

k	p[k]	1	2	3
1	0.08	4	6	6
2	0.08	4	6	8
3	0.12	4	8	6
4	0.12	4	8	8
5	0.08	6	6	6
6	0.08	6	6	8
7	0.12	6	8	6
8	0.12	6	8	8
9	0.04	8	6	6
10	0.04	8	6	8
11	0.06	8	8	6
12	0.06	8	8	8

Benders Decomposition Algorithm

multi-cut version

Iteration #1

Trial X for primal subproblems
(found by minimizing first-stage cost alone)
is

i	Variable	Value
1	X11	2
5	X22	4
9	X33	14

That is, our initial "guess" is to make no first-stage shipments between nodes.

Using the trial first-stage solution, we solve the second-stage problem for each scenario:

```
Scenario #1 with probability
0.08
Optimal objective: 46
 i   variable  value
  5   Y31          2
  6   Y32          2
  9   EX3          4
--------------------
Scenario #2 w/ probability 0.08
Optimal objective: 50
 i   variable  value
  5   Y31          2
  6   Y32          2
  9   EX3          2
--------------------
Scenario #3 w/ probability 0.12
Optimal objective: 80
 i   variable  value
  5   Y31          2
  6   Y32          4
  9   EX3          2
Scenario #4 w/ probability 0.12
Optimal objective: 84
 i   variable  value
  5   Y31          2
  6   Y32          4
--------------------
Scenario #5 w/ probability 0.08
Optimal objective: 74
 i   variable  value
  5   Y31          4
  6   Y32          2
  9   EX3          2
--------------------
Scenario #6 w/ probability 0.08
Optimal objective: 78
 i   variable  value
  5   Y31          4
  6   Y32          2
```

```
Scenario #7 w/ probability 0.12
Optimal objective: 108
 i   variable  value
  5   Y31          4
  6   Y32          4
--------------------
Scenario #8 w/ probability 0.12
Optimal objective: 114
 i   variable  value
  5   Y31          2
  6   Y32          4
 10   SH1          2
--------------------
Scenario #9 w/ probability 0.04
Optimal objective: 102
 i   variable  value
  5   Y31          6
  6   Y32          2
```

```
Scenario #10 w/ probability 0.04
Optimal objective: 108
 i   variable  value
  5   Y31          4
  6   Y32          2
 10   SH1          2
--------------------
Scenario #11 w/ probability 0.06
Optimal objective: 138
 i   variable  value
  5   Y31          4
  6   Y32          4
 10   SH1          2
--------------------
Scenario #12 w/ probability 0.06
Optimal objective: 144
 i   variable  value
  5   Y31          2
  6   Y32          4
 10   SH1          4
```

Primal subproblems summary

Second stage costs:

k	cost	p[k]
1	46	0.08
2	50	0.08
3	80	0.12
4	84	0.12
5	74	0.08
6	78	0.08
7	108	0.12
8	114	0.12
9	102	0.04
10	108	0.04
11	138	0.06
12	144	0.06

First stage cost:	0.00
Expected second stage cost:	91.48
Total:	91.48

*This is an **upper** bound on the optimal solution!*

For each scenario, we also obtain the simplex multipliers (shadow prices, dual variables, etc.) for the three subproblem constraints:

Lagrangian multipliers

i	1	2	3
1	14	17	2
2	14	17	2
3	14	17	2
4	15	18	3
5	14	17	2
6	15	18	3
7	15	18	3
8	15	18	3
9	15	18	3
10	15	18	3
11	15	18	3
12	15	18	3
Sum	176	212	32

These are used to generate a "cut" or linear function which is an approximation (under-estimate) of $Q_k(X)$, the second-stage minimum cost when scenario #k occurs.

The "master problem" then minimizes the sum of the first-stage costs plus the expected value of the (approximation) of second-stage costs:

Solution of Master Problem

i	variable	value
2	X12	2
5	X22	4
8	X32	14

Now node 1 ships all of its supply to node 2!

First-stage cost: 32

k	Q[k]	p[k]
1	¯170	0.08
2	¯166	0.08
3	¯136	0.12
4	¯132	0.12
5	¯142	0.08
6	¯138	0.08
7	¯108	0.12
8	¯102	0.12
9	¯114	0.04
10	¯108	0.04
11	¯78	0.06
12	¯72	0.06

These are the underestimates of the second-stage costs for each scenario, provided by the single cut for each.

Total (estimated) expected value: ¯92.52

Iteration #2

```
Trial X for primal subproblems is
   i   Variable   Value
   2      X12        2
   5      X22        4
   8      X32       14
```

Using this new "trial" first-stage solution, each scenario is evaluated by solving its subproblem:

```
Scenario #1 w/ probability 0.08
Optimal objective: 98
  i   variable  value
  3   Y21          4
  4   Y23          6
  8   EX2          4
--------------------
Scenario #2 w/ probability 0.08
Optimal objective: 136
  i   variable  value
  3   Y21          4
  4   Y23          8
  8   EX2          2
--------------------
Scenario #3 w/ probability 0.12
Optimal objective: 106
  i   variable  value
  3   Y21          4
  4   Y23          6
  8   EX2          2
--------------------
Scenario #4 w/ probability 0.12
Optimal objective: 144
  i   variable  value
  3   Y21          4
  4   Y23          8
--------------------
```

```
Scenario #5 w/ probability 0.08
Optimal objective: 118
  i   variable  value
  3   Y21          6
  4   Y23          6
  8   EX2          2
--------------------
Scenario #6 w/ probability 0.08
Optimal objective: 156
  i   variable  value
  3   Y21          6
  4   Y23          8
--------------------
Scenario #7 w/ probability 0.12
Optimal objective: 126
  i   variable  value
  3   Y21          6
  4   Y23          6
--------------------
Scenario #8 w/ probability 0.12
Optimal objective: 174
  i   variable  value
  3   Y21          4
  4   Y23          8
 10   SH1          2
```

```
Scenario #9 w/ probability 0.04
Optimal objective: 138
  i   variable  value
  3   Y21          8
  4   Y23          6
--------------------
Scenario #10 w/ probability 0.04
Optimal objective: 186
  i   variable  value
  3   Y21          6
  4   Y23          8
 10   SH1          2
--------------------
Scenario #11 w/ probability 0.06
Optimal objective: 156
  i   variable  value
  3   Y21          6
  4   Y23          6
 10   SH1          2
--------------------
Scenario #12 w/ probability 0.06
Optimal objective: 204
  i   variable  value
  3   Y21          4
  4   Y23          8
 10   SH1          4
```

```
Primal subproblems summary
     Second stage costs:
             k   cost   p[k]
             1     98   0.08
             2    136   0.08
             3    106   0.12
             4    144   0.12
             5    118   0.08
             6    156   0.08
             7    126   0.12
             8    174   0.12
             9    138   0.04
            10    186   0.04
            11    156   0.06
            12    204   0.06

First stage cost:                             32.00
Expected second stage cost:                  141.20
                                            -------
Total:                                       173.20
```

This is another upper bound on the optimal expected cost (but not as good as the earlier upper bound!)

Again, we obtain the dual variables of the three constraints from each subproblem solution:

Lagrangian multipliers

i	1	2	3
1	10	4	19
2	10	4	19
3	10	4	19
4	15	9	24
5	10	4	19
6	15	9	24
7	15	9	24
8	15	9	24
9	15	9	24
10	15	9	24
11	15	9	24
12	15	9	24
Sum	160	88	268

These are the *third* set of such dual variables for each scenario, so each Q_k will now be approximated by the maximum of three linear "cuts" or supports.

The master problem is now solved again:

Solution of Master Problem

i	variable	value
1	X11	2.0
5	X22	4.0
8	X32	5.4
9	X33	8.6

First-stage cost: 10.8

k	[k]	p[k]
1	¯35	0.08
2	¯5	0.08
3	¯1	0.12
4	3	0.12
5	¯7	0.08
6	15	0.08
7	27	0.12
8	33	0.12
9	21	0.04
10	45	0.04
11	57	0.06
12	63	0.06

Total (estimated) expected value: 25.52

This is an ***under-estimate*** *of the minimum expected cost!*

...etc.

Iteration #5

```
 Trial X for primal subproblems is
   i   Variable   Value
   1     X11        2
   5     X22        4
   7     X31        2
   8     X32        4
   9     X33        8
```

The master problem in iteration #4 yields the "trial" solution: from node 3, ship 2 units to node 1 and 4 to node 2.

```
Scenario #1 w/ probability 0.08
Optimal objective: ¯12
  i   variable  value
  8   EX2         2
  9   EX3         2
--------------------
Scenario #2 w/ probability 0.08
Optimal objective: ¯8
  i   variable  value
  8   EX2         2
--------------------
Scenario #3 w/ probability 0.12
Optimal objective: ¯4
  i   variable  value
  9   EX3         2
```

```
--------------------
Scenario #4 w/ probability 0.12
Optimal objective: 0
 i   variable  value
--------------------
Scenario #5 w/ probability 0.08
Optimal objective: 8
  i   variable  value
  3   Y21          2
  9   EX3          2
--------------------
Scenario #6 w/ probability 0.08
Optimal objective: 12
  i   variable  value
  3   Y21          2
--------------------
Scenario #7 w/ probability 0.12
Optimal objective: 24
  i   variable  value
  5   Y31          2
--------------------
Scenario #8 w/ probability 0.12
Optimal objective: 30
  i   variable  value
 10   SH1          2
```

```
--------------------
Scenario #9 w/ probability 0.04
Optimal objective: 36
  i   variable  value
  3   Y21          2
  5   Y31          2
--------------------
Scenario #10 w/ probability 0.04
Optimal objective: 42
  i   variable  value
  3   Y21          2
 10   SH1          2
--------------------
Scenario #11 w/ probability 0.06
Optimal objective: 54
  i   variable  value
  5   Y31          2
 10   SH1          2
--------------------
Scenario #12 w/ probability 0.06
Optimal objective: 60
  i   variable  value
 10   SH1          4

Cuts for scenario(s) 3 4 6 7 9 10 11 12 were generated previously!
```

```
--Primal subproblems summary
     Second stage costs:
               k   cost   p[k]
               1    ¯12   0.08
               2     ¯8   0.08
               3     ¯4   0.12
               4      0   0.12
               5      8   0.08
               6     12   0.08
               7     24   0.12
               8     30   0.12
               9     36   0.04
              10     42   0.04
              11     54   0.06
              12     60   0.06

First stage cost:                           14.00
Expected second stage cost:                 15.96
Total:                                      29.96
```

a new upper bound!

Lagrangian multipliers

i	1	2	3
1	4	4	2
2	4	4	14
3	4	10	2
4	14	20	24
5	10	4	2
6	15	9	24
7	15	18	3
8	15	20	25
9	15	9	3
10	15	9	24
11	15	18	3
12	15	20	25
Sum	141	145	151

Solution of Master Problem

```
Optimal value= 29.96
  i   variable  value
  1    X11       2
  5    X22       4
  7    X31       2
  8    X32       4
  9    X33       8

Total (estimated) expected value: 29.96
****************************************
Converged at iteration #5!
X was generated by previous master problem!
```

<>-<>-<>-<>-<>-<>-<>-<>-<>-<>-<>-<>-<>-<>-<>-<>-<>-<>

Optimality Cuts (Multi-cut version)

For each scenario k, a set of linear functions of X has been generated, each of which is an underestimate of the second-stage cost as a function of X.
Hence the maximum provides an underestimating piecewise-linear function of X, each of the form Max { (X)(Lambda[k]) + Alpha[k]}.

Scenario 1

Cut	Lambda									Alpha
1	¯14	¯17	¯2	¯14	¯17	¯2	¯14	¯17	¯2	170
2	¯10	¯4	¯19	¯10	¯4	¯19	¯10	¯4	¯19	178
3	¯10	¯4	¯2	¯10	¯4	¯2	¯10	¯4	¯2	76
4	¯4	¯10	¯2	¯4	¯10	¯2	¯4	¯10	¯2	88
5	¯4	¯4	¯2	¯4	¯4	¯2	¯4	¯4	¯2	52

Scenario 2

Cut	Lambda									Alpha
1	¯14	¯17	¯2	¯14	¯17	¯2	¯14	¯17	¯2	174
2	¯10	¯4	¯19	¯10	¯4	¯19	¯10	¯4	¯19	216
3	¯10	¯4	¯2	¯10	¯4	¯2	¯10	¯4	¯2	80
4	¯4	¯10	¯14	¯4	¯10	¯14	¯4	¯10	¯14	188
5	¯4	¯4	¯14	¯4	¯4	¯14	¯4	¯4	¯14	152

Scenario 3

Cut	Lambda									Alpha
1	¯14	¯17	¯2	¯14	¯17	¯2	¯14	¯17	¯2	204
2	¯10	¯4	¯19	¯10	¯4	¯19	¯10	¯4	¯19	186
3	¯14	¯8	¯2	¯14	¯8	¯2	¯14	¯8	¯2	132
4	¯4	¯10	¯2	¯4	¯10	¯2	¯4	¯10	¯2	108
5	¯4	¯10	¯2	¯4	¯10	¯2	¯4	¯10	¯2	108

Scenario 4

Cut	Lambda									Alpha
1	¯15	¯18	¯3	¯15	¯18	¯3	¯15	¯18	¯3	228
2	¯15	¯9	¯24	¯15	¯9	¯24	¯15	¯9	¯24	324
3	¯15	¯9	¯3	¯15	¯9	¯3	¯15	¯9	¯3	156
4	¯14	¯20	¯24	¯14	¯20	¯24	¯14	¯20	¯24	408
5	¯14	¯20	¯24	¯14	¯20	¯24	¯14	¯20	¯24	408

Scenario 5

Cut	Lambda									Alpha
1	¯14	¯17	¯2	¯14	¯17	¯2	¯14	¯17	¯2	198
2	¯10	¯4	¯19	¯10	¯4	¯19	¯10	¯4	¯19	198
3	¯14	¯8	¯2	¯14	¯8	¯2	¯14	¯8	¯2	144
4	¯4	¯10	¯2	¯4	¯10	¯2	¯4	¯10	¯2	96
5	¯10	¯4	¯2	¯10	¯4	¯2	¯10	¯4	¯2	96

Scenario 6

Cut	Lambda	Alpha

1	¯15	¯18	¯3	¯15	¯18	¯3	¯15	¯18	¯3	222
2	¯15	¯9	¯24	¯15	¯9	¯24	¯15	¯9	¯24	336
3	¯15	¯9	¯3	¯15	¯9	¯3	¯15	¯9	¯3	168
4	¯14	¯20	¯24	¯14	¯20	¯24	¯14	¯20	¯24	396
5	¯15	¯9	¯24	¯15	¯9	¯24	¯15	¯9	¯24	336

Scenario 7

Cut	Lambda									Alpha
1	¯15	¯18	¯3	¯15	¯18	¯3	¯15	¯18	¯3	252
2	¯15	¯9	¯24	¯15	¯9	¯24	¯15	¯9	¯24	306
3	¯15	¯9	¯3	¯15	¯9	¯3	¯15	¯9	¯3	180
4	¯14	¯20	¯5	¯14	¯20	¯5	¯14	¯20	¯5	274
5	¯15	¯18	¯3	¯15	¯18	¯3	¯15	¯18	¯3	252

Scenario 8

Cut	Lambda									Alpha
1	¯15	¯18	¯3	¯15	¯18	¯3	¯15	¯18	¯3	258
2	¯15	¯9	¯24	¯15	¯9	¯24	¯15	¯9	¯24	354
3	¯15	¯9	¯3	¯15	¯9	¯3	¯15	¯9	¯3	186
4	¯14	¯20	¯24	¯14	¯20	¯24	¯14	¯20	¯24	436
5	¯15	¯20	¯25	¯15	¯20	¯25	¯15	¯20	¯25	450

Scenario 9

Cut	Lambda									Alpha
1	¯15	¯18	¯3	¯15	¯18	¯3	¯15	¯18	¯3	246

```
   2  ¯15 ¯9 ¯24 ¯15 ¯9 ¯24 ¯15 ¯9 ¯24    318
   3      ¯15 ¯9 ¯3 ¯15 ¯9 ¯3 ¯15 ¯9 ¯3    192
   4  ¯14 ¯20 ¯5 ¯14 ¯20 ¯5 ¯14 ¯20 ¯5    262
   5      ¯15 ¯9 ¯3 ¯15 ¯9 ¯3 ¯15 ¯9 ¯3    192
----------------------------------------
Scenario 10
 Cut         Lambda                              Alpha
   1      ¯15 ¯18 ¯3 ¯15 ¯18 ¯3 ¯15 ¯18 ¯3     252
   2      ¯15 ¯9 ¯24 ¯15 ¯9 ¯24 ¯15 ¯9 ¯24     366
   3         ¯15 ¯9 ¯3 ¯15 ¯9 ¯3 ¯15 ¯9 ¯3     198
   4  ¯15 ¯20 ¯25 ¯15 ¯20 ¯25 ¯15 ¯20 ¯25     440
   5      ¯15 ¯9 ¯24 ¯15 ¯9 ¯24 ¯15 ¯9 ¯24     366
----------------------------------------
Scenario 11
 Cut         Lambda                           Alpha
   1  ¯15 ¯18 ¯3 ¯15 ¯18 ¯3 ¯15 ¯18 ¯3     282
   2  ¯15 ¯9 ¯24 ¯15 ¯9 ¯24 ¯15 ¯9 ¯24     336
   3      ¯15 ¯9 ¯3 ¯15 ¯9 ¯3 ¯15 ¯9 ¯3     210
   4  ¯14 ¯20 ¯5 ¯14 ¯20 ¯5 ¯14 ¯20 ¯5     302
   5  ¯15 ¯18 ¯3 ¯15 ¯18 ¯3 ¯15 ¯18 ¯3     282
----------------------------------------
Scenario 12
 Cut         Lambda                              Alpha
   1      ¯15 ¯18 ¯3 ¯15 ¯18 ¯3 ¯15 ¯18 ¯3     288
   2      ¯15 ¯9 ¯24 ¯15 ¯9 ¯24 ¯15 ¯9 ¯24     384
```

3	¯15	¯9	¯3	¯15	¯9	¯3	¯15	¯9	¯3	216
4	¯15	¯20	¯25	¯15	¯20	¯25	¯15	¯20	¯25	480
5	¯15	¯20	¯25	¯15	¯20	¯25	¯15	¯20	¯25	480

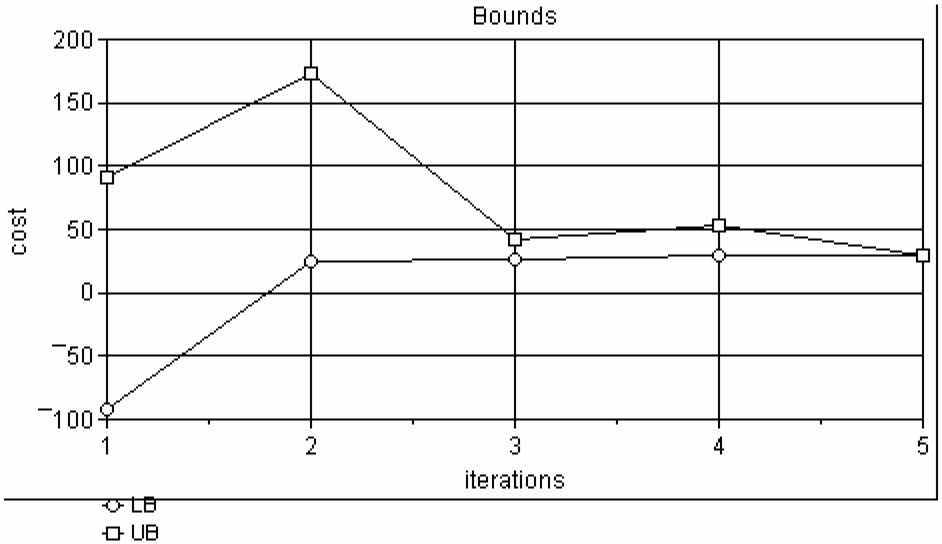
Best Solution of Benders

Total cost: 29.96, found at iteration #5
Best lower bound: 29.96

Gap= ¯3.5527137E¯15, or ¯1.185819E¯14%

Non-zero Stage One Variables:

i	variable	value
1	X11	2
5	X22	4
7	X31	2
8	X32	4
9	X33	8


Bounds
200
150
100
50
0
−50
−100
cost
1
2
3
4
5
iterations
LB
UB

Certainty-Equivalent Tableau

Solving the LP problem assuming the demands are certain to be their expected values:

b	z	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9	10	11	12
0	1	0	2	3	2	0	2	3	2	0	6	10	6	15	12	15	¯4	¯4	¯2	15	20	30
2	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0
5.6	0	1	0	0	1	0	0	1	0	0	¯1	¯1	1	0	1	0	¯1	0	0	1	0	0
7.2	0	0	1	0	0	1	0	0	1	0	1	0	¯1	¯1	0	1	0	¯1	0	0	1	0
7	0	0	0	1	0	0	1	0	0	1	0	1	0	1	¯1	¯1	0	0	¯1	0	0	1

Optimal Solution

Found by solving ***certainty equivalent problem***,
i.e., replacing all random parameters by their expected values.

Total objective function: 16.8
Stage One: nonzero variables:

i	variable	value
1	X11	2.0
5	X22	4.0
7	X31	3.6
8	X32	3.4
9	X33	7.0

Second Stage: nonzero variables

i	variable	value
8	EX2	0.2

The optimal cost ($16.80) of this LP is not the *actual* expected cost of this first-stage shipment plan!
To evaluate the expected cost, we must solve each scenario's subproblem, using this shipment plan as "trial" solution.

Evaluation of trial solution

i	variable	X[i]
1	X11	2.0
5	X22	4.0
7	X31	3.6
8	X32	3.4
9	X33	7.0

Summary

Second stage objective:

k	objective	p[k]
1	-14.0	0.08
2	2.0	0.08
3	-2.4	0.12
4	13.6	0.12
5	-3.6	0.08
6	17.4	0.08
7	13.8	0.12
8	43.0	0.12
9	20.4	0.04
10	47.4	0.04
11	43.8	0.06
12	73.0	0.06

```
First stage objective:                    17.60
Expected second stage objective:          18.02
Total:                                    35.62
```

If we were to use this shipment plan, our expected cost would exceed the minimum ($26.96) by

$$\$35.62 - 26.96 = \$\ 8.66 = \text{VSS}\ \textit{(value of stochastic solution)}\,!$$

Suppose that we had "perfect information" about the demands <u>before</u> choosing the first-stage variables:

Optimization with perfect information

```
-------------------------------------------------------
Solution for scenario #1          Solution for scenario #2

Optimal cost: 2                   Optimal cost: 6
Stage One: nonzero variables:     Stage One: nonzero variables:
 i         value Name              i         value Name
 1         2.00  X11               1         2.00 X11
 5         4.00  X22               5         4.00 X22
 7         2.00  X31               7         2.00 X31
 8         6.00  X32               8         4.00 X32
 9         6.00  X33               9         8.00 X33

Second-stage: nonzero variables   Second-stage: nonzero variables
 i         value Name              i         value Name
 8         4.00  EX2               8         2.00  EX2
-------------------------------   -------------------------------
```

Solution for scenario #3

Optimal cost: 10

Stage One: nonzero variables:

i	value	Name
1	2.00	X11
5	4.00	X22
7	2.00	X31
8	6.00	X32
9	6.00	X33

Second-stage: nonzero variables

i	value	Name
8	2.00	EX2

Solution for scenario #4

Optimal cost: 14

Stage One: nonzero variables:

i	value	Name
1	2.00	X11
5	4.00	X22
7	2.00	X31
8	4.00	X32
9	8.00	X33

Second-stage: nonzero variables

(none)

Solution for scenario #5

Optimal cost: 12

Stage One: nonzero variables:

i	value	Name
1	2.00	X11
5	4.00	X22
7	4.00	X31
8	4.00	X32
9	6.00	X33

Second-stage: nonzero variables

i	value	Name
8	2.00	EX2

Solution for scenario #6

Optimal cost: 16

Stage One: nonzero variables:

i	value	Name
1	2.00	X11
5	4.00	X22
7	4.00	X31
8	2.00	X32
9	8.00	X33

Second-stage: nonzero variables

Solution for scenario #7

Optimal cost: 20

Stage One: nonzero variables:

i	value	Name
1	2.00	X11
5	4.00	X22
7	4.00	X31
8	4.00	X32
9	6.00	X33

Second-stage: nonzero variables:

(none)

Solution for scenario #8

Optimal cost: 44

Stage One: nonzero variables:

i	value	Name
1	2.00	X11
5	4.00	X22
7	2.00	X31
8	4.00	X32
9	8.00	X33

Second-stage: nonzero variables

i	value	Name
10	2.00	SH1

Solution for scenario #9

Optimal cost: 22

Stage One: nonzero variables:

i	value	Name
1	2.00	X11
5	4.00	X22
7	6.00	X31
8	2.00	X32
9	6.00	X33

Second-stage: nonzero variables:

(none)

Solution for scenario #10

Optimal cost: 46

Stage One: nonzero variables:

i	value	Name
1	2.00	X11
5	4.00	X22
7	4.00	X31
8	2.00	X32
9	8.00	X33

Second-stage: nonzero variables

i	value	Name
10	2.00	SH1

Solution for scenario #11

Optimal cost: 50

Stage One: nonzero variables:

i	value	Name
1	2.00	X11
5	4.00	X22
7	4.00	X31
8	4.00	X32
9	6.00	X33

Second-stage: nonzero variables

i	value	Name
10	2.00	SH1

Solution for scenario #12

Optimal cost: 74

Stage One: nonzero variables:

i	value	Name
1	2.00	X11
5	4.00	X22
7	2.00	X31
8	4.00	X32
9	8.00	X33

Second-stage: nonzero variables

i	value	Name
10	4.00	SH1

Expected cost with perfect information: 23.6

The **EVPI** *(expected value of perfect information)* is therefore the difference between the expected value *with* perfect information and the expected value *without* such information, i.e.,

$$\text{EVPI} = \$26.96 - \$23.60 = \$6.36$$