

**EXAMPLE**

Set Coverage Matrix (A)

	1	2	3	4	5	6	7	8	9	0	1	2	3	4	5	6	7	8	9	0	1	2	3	4	5
1	0	1	0	0	0	0	1	0	0	0	1	0	1	0	1	0	1	1	0	0	0	0	0	0	0
2	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	1	0	1
3	0	1	1	1	0	0	1	0	0	0	0	0	0	1	0	0	0	0	1	0	0	1	0	0	0
4	0	1	0	0	0	0	1	0	1	0	0	0	0	0	0	1	0	0	0	0	1	0	1	0	1
5	0	0	0	0	0	0	1	0	1	1	1	0	0	0	0	0	1	0	0	0	0	1	0	1	1
6	0	0	0	1	0	0	0	1	0	0	0	0	0	1	0	1	1	0	0	1	0	0	1	1	0
7	0	1	0	1	1	0	1	0	0	0	0	1	0	0	1	1	0	1	1	0	0	1	1	1	0
8	1	0	0	0	0	1	1	0	0	1	0	0	1	1	0	0	0	1	0	1	1	1	0	1	1
C:	2	3	3	4	4	4	5	5	5	5	6	7	7	7	7	1	1	1	1	1	1	1	1	1	2

Density = 34.00%



author



## **Initial values of Lagrangian multipliers**

For point  $i$ , choose the value

- zero
- ✓ ● smallest of costs of sets covering point  $i$
- average of costs of sets covering point  $i$

## Iteration # 1

Current multipliers:

i	1	2	3	4	5	6	7	8
w[i]	3	2	3	3	5	4	3	2

Solving Lagrangian relaxation:

Set(s) 23 24 25 removed from problem  
(P= 16 17 22 > 14 = incumbent!)

# sets remaining is 22

\*\*\* Dual value is -20 \*\*\*

Sets in cover: # 1 2 3 4 7 8 9 10 11 13 14 16 19 22

Points not covered:

(none)

### Heuristic Adjustment

```
n_take= 1, c_type= 1 row selection: smallest
      Try removing sets from cover:
      set      1   2   3   4   7   8   9   10  11  13  14  16  19  22
      Set Cost   2   3   3   4   5   5   5   5   5   7   7   11  12  14
      Reduced cost -2 -11 -2 -6 -11 -2 -3 -2  0 -2 -1 -2 -1  0
      Set 22 can be removed!
      Set 19 can be removed!
      Set 16 can be removed!
      Set 14 can be removed!
      Set 13 can be removed!
      Set 11 can be removed!
      Set 10 can be removed!
      Set 9 can be removed!
      Set 8 can be removed!
      Set 3 can be removed!
      Set 1 can be removed!
      Covering sets are 2 4 7
      Heuristic solution cost is 12
```

\*\*\* New incumbent! \*\*\* Cover is 2 4 7  
with cost 12

Subgradient of Dual Objective:

+1 for points:

$\leq -2$  for points: 1 2 3 4 5 6 7 8

Norm squared is 156

Stepsize is 0.205128

## Iteration # 2

Current multipliers:

i	1	2	3	4	5	6	7	8
w[i]	2.179	1.589	1.769	2.384	4.179	2.974	1.974	0.9743

Solving Lagrangian relaxation:

```
Set(s) 22 removed from problem
(P= 14 > 12 = incumbent!)
# sets remaining is 21
*** Dual value is -0.666667 ***
Sets in cover: # 1 2 3 4 7 8 9 10
Points not covered:
          (none)
```

## Computation of $P_{22}$

Since  $X_{22}=0$  in the solution of the Lagrangian relaxation,  $P_{22} = \Phi(\lambda) + \bar{C}_{22}$  where  $\bar{C}_{22}$  is the reduced cost of  $X_{22}$ , namely

$$\begin{aligned}\bar{C}_{22} &= C_{22} - [\lambda_3 + \lambda_5 + \lambda_6 + \lambda_8] \\ &= 14 - [1.769 + 4.179 + 2.974 + 0.9743] \\ &= 14 - 9.896 = 4.1037\end{aligned}$$

$$\Phi(\lambda) = -0.66667$$

A <sup>22</sup>	
1	0
2	0
3	1
4	0
5	1
6	1
7	0
8	1
C:	
	1
	4

Heuristic Adjustment

n\_take= 1, c\_type= 1 row selection: smallest

Try removing sets from cover:

set	1	2	3	4	7
Set Cost	2	3	3	4	5
Reduced cost	-0.564103	-6.89744	-0.358974	-2.71795	-6.28205

8	9	10
5	5	5
-0.153846	-1.5641	-0.153846

Set 10 can be removed!

Set 9 can be removed!

Set 8 can be removed!

Set 3 can be removed!

Set 1 can be removed!

Covering sets are 2 4 7

Heuristic solution cost is 12

Subgradient of Dual Objective:  
+1 for points:  
-1 for points: 1 6  
 $\leq -2$  for points: 2 3 4 5 7 8  
Norm squared is 31  
Stepsize is 0.408602

## Iteration # 3

Current multipliers:

i	1	2	3	4	5	6	7	8
w[i]	1.770	0.7725	0.5434	1.567	3.362	2.565	1.157	0.1571

Solving Lagrangian relaxation:

Set(s) 17 18 19 20 21 removed from problem  
(P= 13.512 16.1034 12.9942 16.9446 16.5343 > 12=incumbent!)

# sets remaining is 16  
\*\*\* Dual value is 7.03143 \*\*\* (Improvement: 7.6981)

Sets in cover: # 2 4 7

Points not covered:  
(none)

### Heuristic Adjustment

```
n_take= 1, c_type= 1 row selection: smallest
      Try removing sets from cover:
      set      2      4      7
      Set Cost  3      4      5
      Reduced cost -2.81141 -0.266336 -1.78743
Covering sets are 2 4 7
Heuristic solution cost is 12
```

### Subgradient of Dual Objective:

```
+1 for points:
-1 for points: 4
≤ -2 for points: 3 7
Norm squared is 9
Stepsize is 0.552063
```

## Iteration # 4

Current multipliers:

i	1	2	3	4	5	6	7	8
w[i]	1.770	0.7725	0	1.015	3.362	2.565	0.05302	0.1571

Solving Lagrangian relaxation:

```
Set(s) 5 6 12 13 14 15 16 removed from problem
(P= 13.0322 12.928 13.3143 13.3093 14.1572 13.5194 14.6802
 > 12 = incumbent!)
# sets remaining is 9
*** Dual value is 9.08519 *** (Improvement: 2.05376)
```

```
Sets in cover: # 2
Points not covered: 5 6 8
```

Heuristic Adjustment

n\_take= 1, c\_type= 1,

row selection rule: row with maximum multiplier

Add set 7 to cover row 5

Add set 4 to cover row 6

Try removing sets from cover:

set	2	4	7
-----	---	---	---

Set Cost	3	4	5
----------	---	---	---

Reduced cost -0.6118 1.381 0.4122

Covering sets are 2 4 7

Heuristic solution cost is 12

Subgradient of Dual Objective:

+1 for points: 5 6 8

Norm squared is 3

Stepsize is 0.9716

## Iteration # 5

Current multipliers:

i	1	2	3	4	5	6	7	8
w[i]	1.770	0.7725	0	1.015	4.333	3.537	0.05302	1.128

Solving Lagrangian relaxation:

\*\*\* Dual value is 9.34886 \*\*\* (Improvement: 0.263671)

Sets in cover: # 2 7 8 9 10

Points not covered: (none)

Heuristic Adjustment

```
n_take= 1, c_type= 1 row selection: smallest
      Try removing sets from cover:
set      2      7      8      9      10
Set Cost  3      5      5      5      5
Reduced cost -0.6118 -1.53102 -0.308244 -0.349233 -0.462641
      Set 10 can be removed!
      Set 9 can be removed!
      Covering sets are 2 7 8
      Heuristic solution cost is 13
```

## Subgradient of Dual Objective:

```
+1 for points:
-1 for points: 1 7 8
≤ -2 for points: 4 5
Norm squared is 11
Stepsize is 0.241012
```

## Iteration # 6

Current multipliers:

i	1	2	3	4	5	6	7	8
w[ii]	1.53	0.7725	0	0.5333	3.852	3.537	0	0.8877

Solving Lagrangian relaxation:

Set(s) 3 removed from problem  
(P= 13 > 12 = incumbent!)

# sets remaining is 8  
\*\*\* Dual value is 10.77 \*\*\* (Improvement: 1.424)

Sets in cover: # 7 8

Points not covered: 2

Heuristic Adjustment

n\_take= 1, c\_type= 1,  
row selection rule: row with maximum multiplier

Add set 1 to cover row 2  
Try removing sets from cover:  
set 1 7 8  
Set Cost 2 5 5  
Reduced cost 0.3397 -0.2729 -0.06723

Covering sets are 1 7 8  
Heuristic solution cost is 12

Subgradient of Dual Objective:  
+1 for points: 2  
Norm squared is 1  
Stepsize is 1.227

## Iteration # 10

Current multipliers:

i	1	2	3	4	5	6	7	8
w[i]	1.243	1.651	0.07705	0.1848	4.053	3.739	0.07705	0.5392

Solving Lagrangian relaxation:

Set(s) 11 removed from problem

(P= 12.09 &gt; 12 = incumbent!)

# sets remaining is 7

\*\*\* Dual value is 11.14 \*\*\* (Improvement: 0.5077)

Sets in cover: # 1 2

Points not covered: 5 6

Converged!

Solution is sets # 2 4 7  
with value 12

Since all costs of example are integer-valued, the lower bound 11.14 may be rounded up to 12, which is the value of the incumbent!

