EXAMPLE

Set Covering-4

Set Coverage Matrix (A)

 \bigotimes

____ Density = 34.00%



Initial values of Lagrangian multipliers

For point i, choose the value

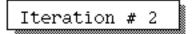
- zero
- smallest of costs of sets covering point i
 - average of costs of sets covering point i

	i	1	2	3	4	5	6	7	8	
w[i	ונ	3	2	3	3	5	4	3	2	

Solving Lagrangian relaxation:

```
n_take= 1, c_type= 1 row selection: smallest
     Try removing sets from cover:
                                      9
-5
-3
                                         10
5
-2
                                             set
                         3
3
                                   8
5
-2
                      2
                            4
                                                             22
                                 7
                 1
                 2
                      3
                            4
                                 5
  Set Cost
                                                             14
                        -2
                            ĥ
                               -11
  Reduced cost
                 -2
                    -11
                                                               Û.
     Set 22 can be removed!
     Set.
         19 can be removed!
     Set.
         16 can be removed!
     Set
         14 can be removed!
         13 can be removed!
     Set.
     Set.
         11 can be removed!
     Set
         10 can be removed!
     Set
         9 can be removed!
     Set.
         8
            can be removed!
         3
     Set.
            can be removed!
     Set 1
            can be removed!
     Covering sets are 2
                           4
     Heuristic solution cost is 12
```

```
*** New incumbent! *** Cover is 2 4 7
with cost 12
Subgradient of Dual Objective:
   +1 for points:
    ≤ -2 for points: 1 2 3 4 5 6 7 8
   Norm squared is 156
   Stepsize is 0.205128
```



i 1 2 3 4 5 6 7 8 w[i] 2.179 1.589 1.769 2.384 4.179 2.974 1.974 0.9743

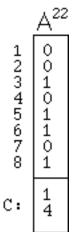
Solving Lagrangian relaxation:

Computation of $\mathsf{P}_{\!\scriptscriptstyle 22}$

Since X₂₂=0 in the solution of the Lagrangian relaxation, P₂₂ = $\Phi(\lambda)$ + \overline{C}_{22} where \overline{C}_{22} is the reduced cost of X₂₂, namely

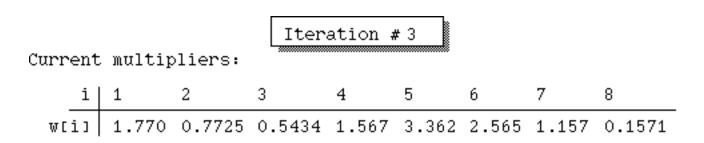
$$\overline{C}_{22} = C_{22} - [\lambda_3 + \lambda_5 + \lambda_6 + \lambda_8] = 14 - [1.769 + 4.179 + 2.974 + 0.9743] = 14 - 9.896 = 4.1037$$

$$\Phi(\boldsymbol{\lambda}) = -0.66667$$



```
n_take= 1, c_type= 1 row selection: smallest
     Try removing sets from cover:
set
                          2
                                    3
                                               4
                          3
Set Cost
               2
                                    3
Reduced cost -0.564103
                        -6.89744
                                              -2.71795
                                   -0.358974
                                                       -6.28205
                             9
                  8
                                     10
                  5
                             5
                                      5
                                     -0.153846
                 -0.153846
                            -1.5641
     Set 10 can be removed!
     Set 9 can be removed!
     Set 8 can be removed!
     Set 3 can be removed!
     Set 1 can be removed!
     Covering sets are 2
                             7
     Heuristic solution cost is 12
```

```
Subgradient of Dual Objective:
+1 for points:
-1 for points: 1 6
≤ 72 for points: 2 3 4 5 7 8
Norm squared is 31
Stepsize is 0.408602
```



Solving Lagrangian relaxation:

n_take= 1, c_type= 1 row selection: smallest Try removing sets from cover: set. 2 7 4 3 4 Set Cost Reduced cost-2.81141 -0.266336 -1.78743 Covering sets are 2 4 7 Heuristic solution cost is 12 Subgradient of Dual Objective: +1 for points: -1 for points: 4 ≤ 72 for points: 3 7 Norm squared is 9 Stepsize is 0.552063

Iteration #4

Current multipliers:

i 1 2 3 4 5 6 7 8 w[i] 1.770 0.7725 0 1.015 3.362 2.565 0.05302 0.1571

Solving Lagrangian relaxation:

```
Set(s) 5 6 12 13 14 15 16 removed from problem
(P= 13.0322 12.928 13.3143 13.3093 14.1572 13.5194 14.6802
                        > 12 = incumbent!)
# sets remaining is 9
*** Dual value is 9.08519 *** (Improvement: 2.05376)
Sets in cover: # 2
Points not covered: 5 6 8
```

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Heuristic Adjustment

```
n_take= 1, c_type= 1,
row selection rule: row with maximum multiplier
     Add set 7 to cover row 5
     Add set 4 to cover row 6
     Try removing sets from cover:
                            2
              set.
                                         7
                                   4
                            3
              Set Cost
              Reduced cost-0.6118 1.381 0.4122
     Covering sets are 2 4 7
     Heuristic solution cost is 12
Subgradient of Dual Objective:
   +1 for points: 5 6 8
   Norm squared is 3
   Stepsize is 0.9716
```

i	1	2	3	4	5	6	7	8
w[i]	1.770	0.7725	0	1.015	4.333	3.537	0.05302	1.128

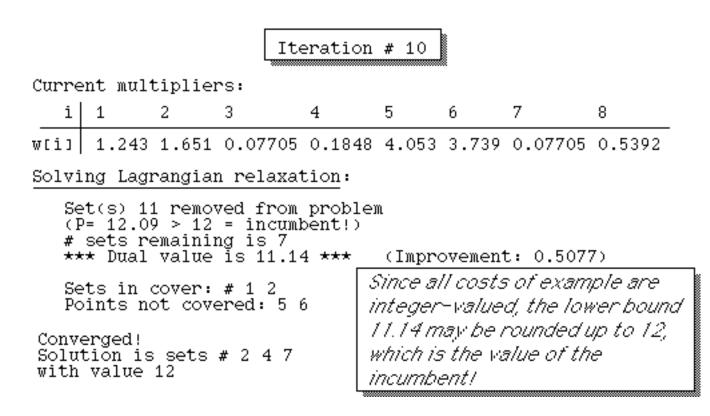
Solving Lagrangian relaxation:

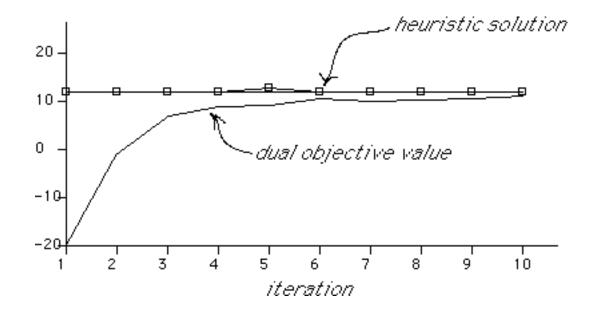
```
n_take= 1, c_type= 1 row selection: smallest
     Try removing sets from cover:
                                           9
set.
                       7
                                 8
                                                     10
                                 5
                                           5
Set Cost
               3
                       5
                                                      5
Reduced cost -0.6118
                      -1.53102
                               -0.308244 -0.349233 -0.462641
     Set 10 can be removed!
     Set 9 can be removed!
     Covering sets are 2 7 8
     Heuristic solution cost is 13
Subgradient of Dual Objective:
   +1 for points:
   -1 for points: 1 7 8
   ≤ 72 for points: 4 5
   Norm squared is 11
   Stepsize is 0.241012
```

Solving Lagrangian relaxation:

```
Set(s) 3 removed from problem
(P= 13 > 12 = incumbent!)
# sets remaining is 8
*** Dual value is 10.77 *** (Improvement: 1.424)
Sets in cover: # 7 8
Points not covered: 2
```

```
n_take= 1, c_type= 1,
row selection rule: row with maximum multiplier
     Add set 1 to cover row 2
     Try removing sets from cover:
            set
                         1
                                  7
                                          8
                                 5
                                          5
                         2
            Set Cost
            Reduced cost 0.3397 -0.2729 -0.06723
     Covering sets are 1 7 8
     Heuristic solution cost is 12
Subgradient of Dual Objective:
   +1 for points: 2
   Norm squared is 1
   Stepsize is 1.227
```





@D.L.Bricker, U. of IA, 1998