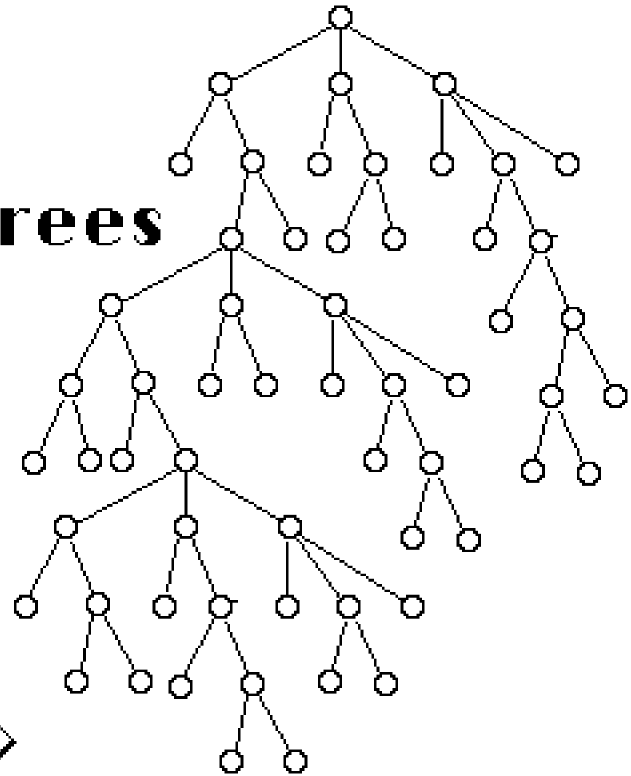
Search Trees



This Hypercard stack was prepared by:
Dennis L. Bricker,
Dept. of Industrial Engineering,
University of Iowa,
Iowa City, Iowa 52242
e-mail: dennis-bricker@uiowa.edu

author

Search Trees

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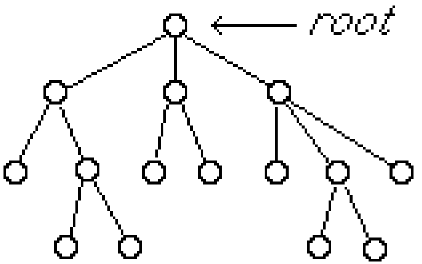


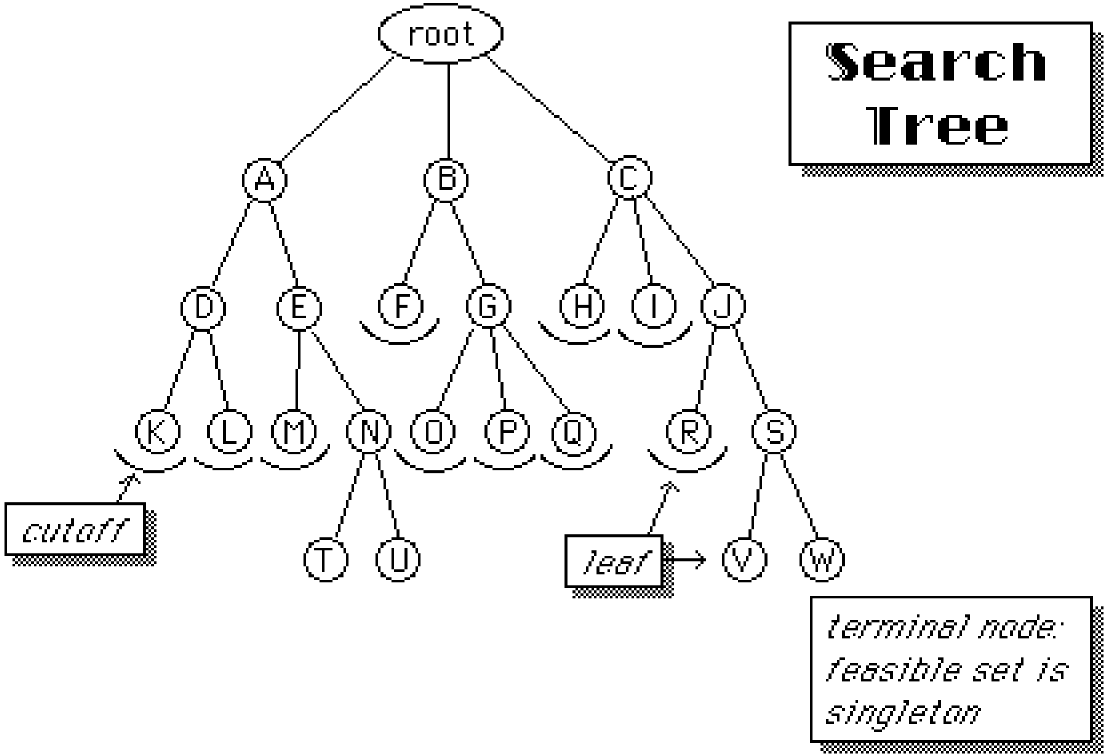
- Each node of the **search tree** for a problem represents a **subset of feasible solutions** of the problem
- The **root** of the tree represents the set of all feasible solutions of the problem
- The **descendents** of each node of the tree represent a **partition** of the set represented by that node

A collection of subsets B_i of set A $(i=1,2,\ldots t)$ is a **partition** if

$$B_1 \cup B_2 \cup B_3 \cdots \cup B_t = A$$

and

$$B_i \cap B_j = \emptyset \quad \text{if } i \neq j$$


root
A
B
C
D
E
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H
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O
P
Q
R
S
T
U
V
W
cutoff
leaf
Search Tree
terminal node: feasible set is singleton

Example: Ranking Nodes in a Preference Graph

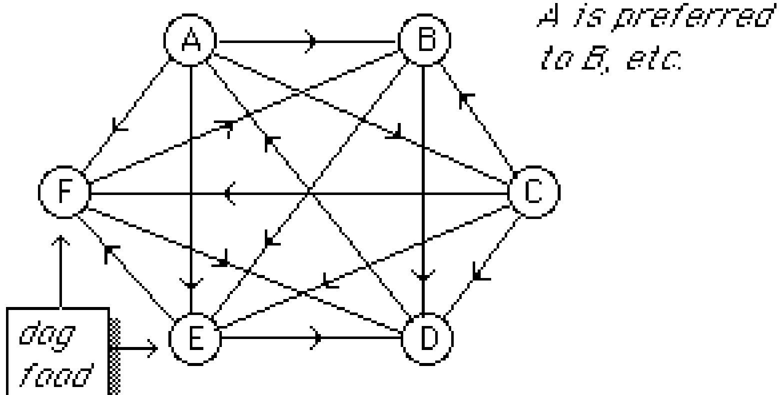
In many experiments (especially in the social sciences, when numerical measurement of attributes are difficult or impossible), one is required to **rank** a set of objects by comparing only **two at a time**.

Example

Six different dog foods are to be ranked according to their appeal to dogs.

Each day, 2 of the 6 are served to a dog, who indicates his preference by finishing it first.

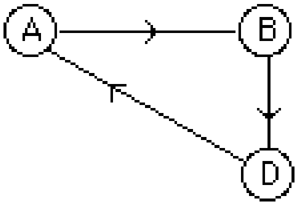
Preference Graph



Preference Matrix

	A	B	C	D	E	F
A	–	1	1	0	1	1
B	0	–	0	1	1	0
C	0	1	–	1	1	1
D	1	0	0	–	0	0
E	0	0	0	1	–	1
F	0	1	0	1	0	–

In the dog food example, the dog exhibited some inconsistency: for example,



he preferred A over B,
B over D,
and D over A!

How can we establish a "good" ranking?

Methods for Ranking

- ranking by score: the score of an object is the number of pairs in which it is preferred (i.e., the row-sum of the preference matrix).
  — ties may occur
  — assumes every possible pair was compared

	A	B	C	D	E	F	***score***
A	-	1	1	0	1	1	**4**
B	0	-	0	1	1	0	**2**
C	0	1	-	1	1	1	**4**
D	1	0	0	-	0	0	**1**
E	0	0	0	1	-	1	**2**
F	0	1	0	1	0	-	**2**

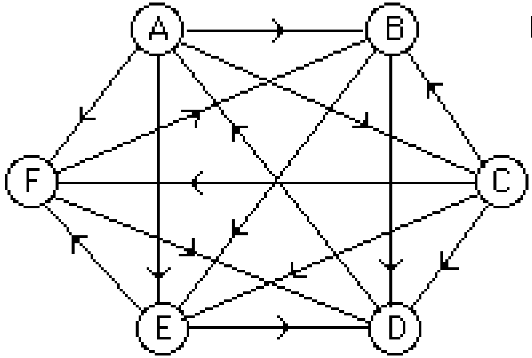
For example,
A > C > B > E > F > D
or C > A > F > E > B > D
etc.

Methods for Ranking

- **ranking by Hamiltonian path:** find a path through every node of the preference graph such that each node is preferred over its successor.

  For example, A→C→B→E→F→D

  or A→C→E→F→B→D

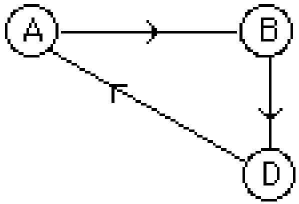


(several such paths may exist!)

Methods for Ranking

- **ranking with minimum discrepancies**

  A discrepancy is an instance in which
  X is ranked above Y, but Y is preferred to X



For example, the ranking A > B > D has one discrepancy (i.e., A>D)

— does not assume that every pair was compared!

— is a difficult problem to solve

Using a Search Tree for Minimum Discrepancy Ranking

Two different methods for partitioning:

- choose a pair of objects X & Y which have not been ranked.
  Form two subsets of rankings:
  - --those in which X > Y, i.e., X is ranked above Y
  - --those in which Y > X, i.e., Y is ranked above X

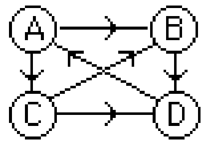
Second method of partitioning:

- an object is assigned to a position in the ranking

e.g., in the first partition, **n** nodes are created, in each of which one of the **n** objects is assigned to the **first** position in the ranking, and

in the second partition, **n−1** nodes are created, one for each of the remaining **n−1** objects which might be assigned to the **second** position in the ranking, etc.

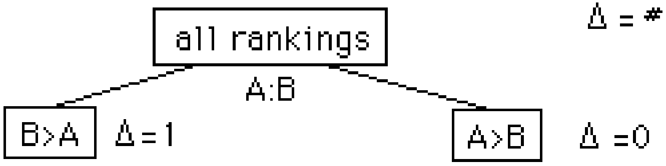
Example



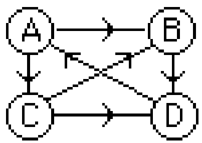
	A	B	C	D	score
A	-	1	1	0	2
B	0	-	0	1	1
C	0	1	-	1	2
D	1	0	0	-	1

Δ = # discrepancies

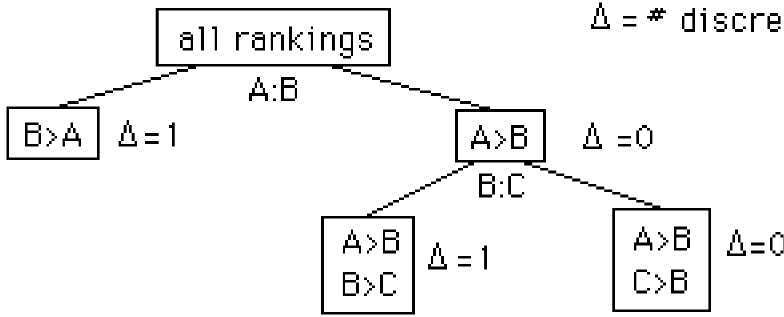
First Partitioning Method



We will partition the most promising node, that with no discrepancies



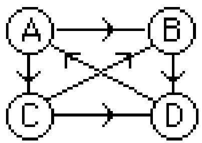
Δ = # discrepancies

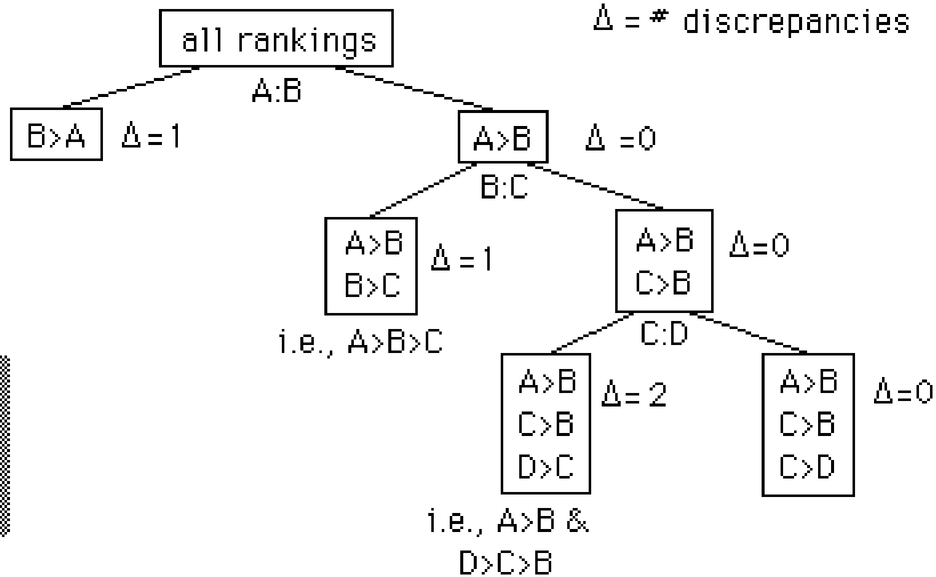


	A	B	C	D
A	–	1	1	0
B	0	–	0	1
C	0	1	–	1
D	1	0	0	–

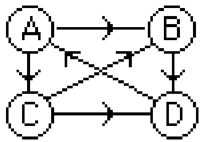
i.e., A>B>C
(B>C is a discrepancy)

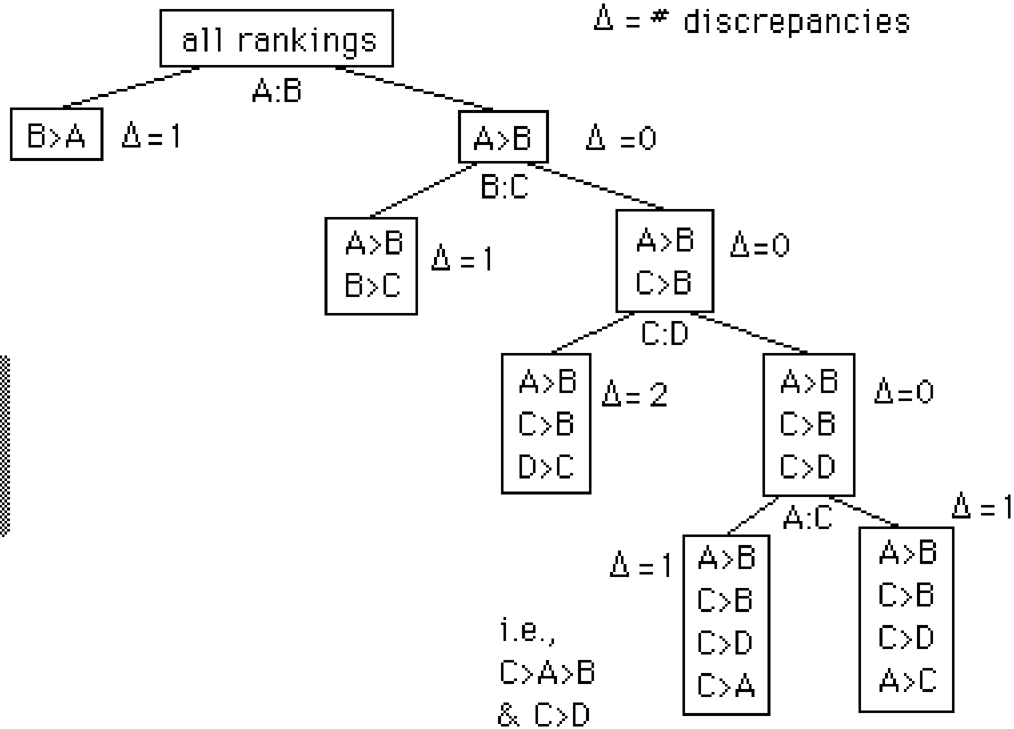
Again, we partition the most promising node



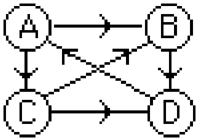


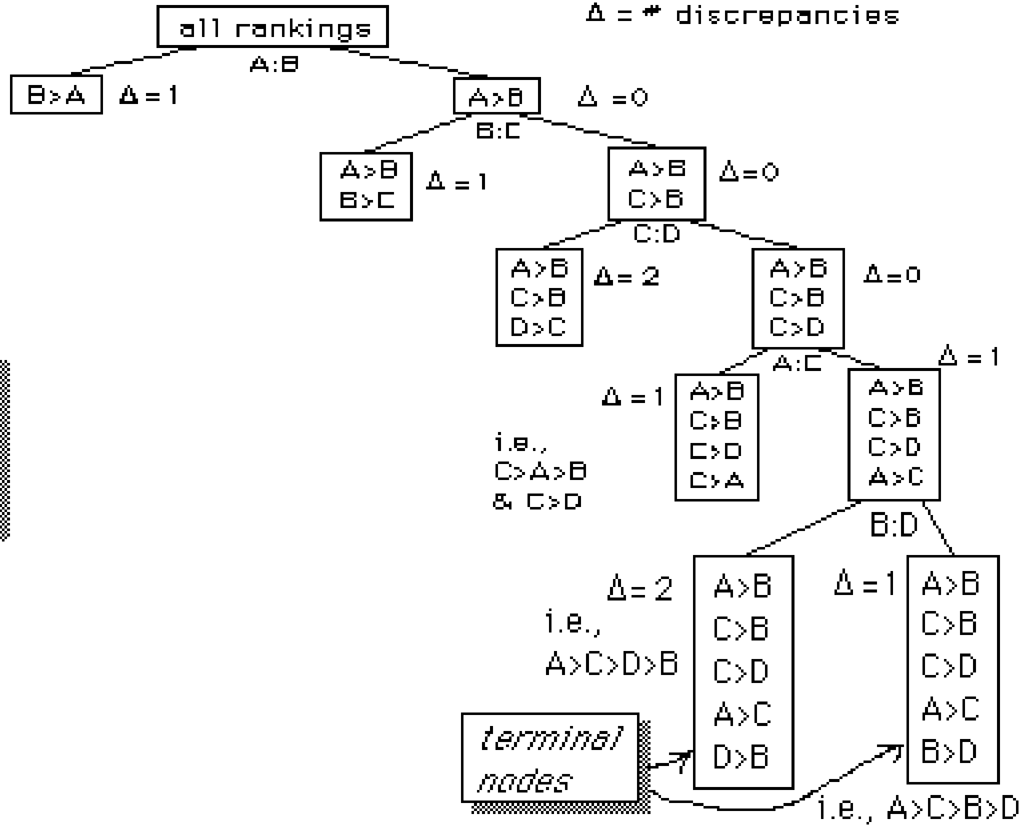
	A	B	C	D
A	–	1	1	0
B	0	–	0	1
C	0	1	–	1
D	1	0	0	–



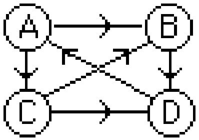


	A	B	C	D
A	–	1	1	0
B	0	–	0	1
C	0	1	–	1
D	1	0	0	–

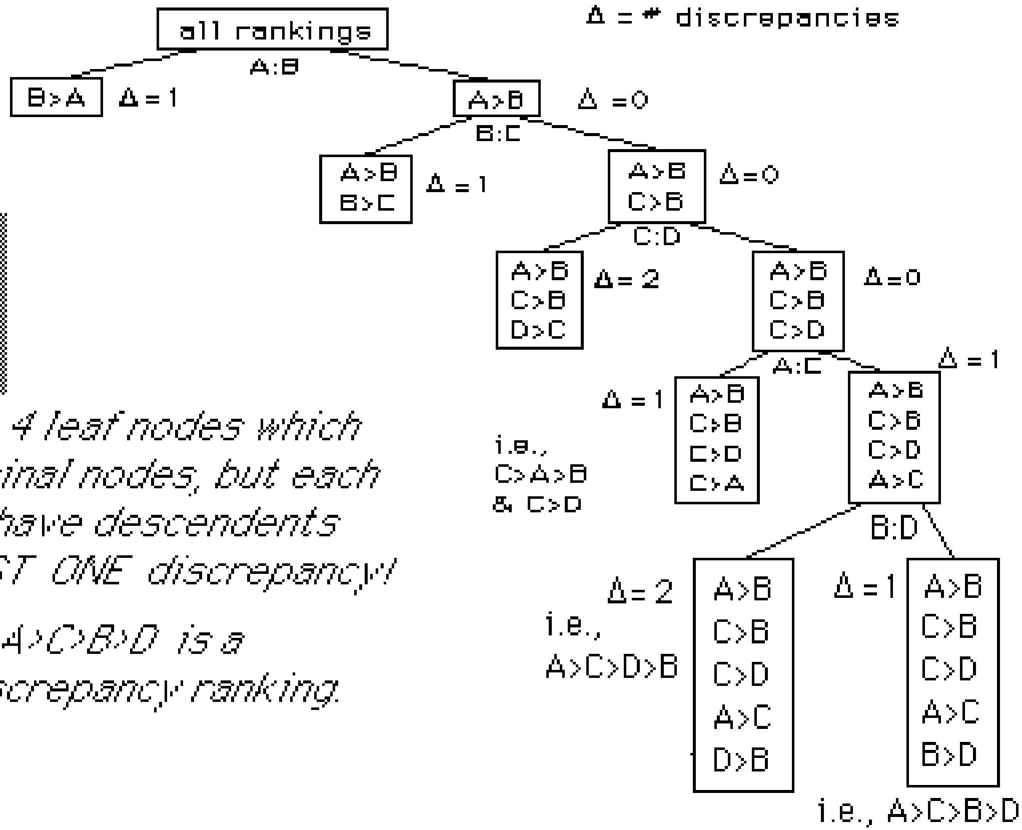




	A	B	C	D
A	-	1	1	0
B	0	-	0	1
C	0	1	-	1
D	1	0	0	-



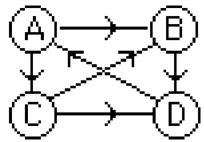
	A	B	C	D
A	-	1	1	0
B	0	-	0	1
C	0	1	-	1
D	1	0	0	-



There remain 4 leaf nodes which are NOT terminal nodes, but each of these will have descendents with AT LEAST ONE discrepancy!

The ranking A>C>B>D is a minimum-discrepancy ranking.

Example

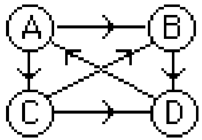


Second Partitioning Method

	A	B	C	D	score
A	-	1	1	0	2
B	0	-	0	1	1
C	0	1	-	1	2
D	1	0	0	-	1

Δ = # discrepancies

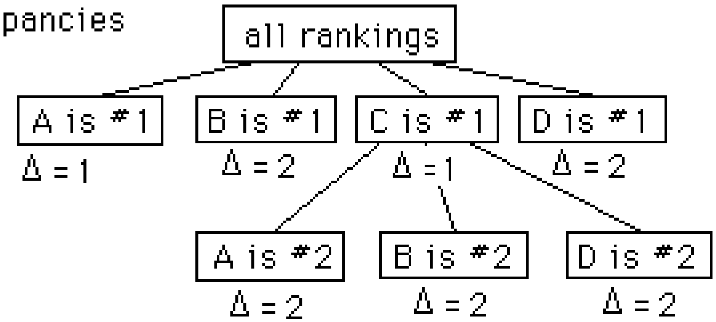
We will partition the most promising node, that with one discrepancy



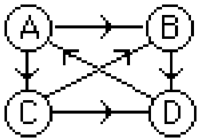
	A	B	C	D
A	–	1	1	0
B	0	–	0	1
C	0	1	–	1
D	1	0	0	–

Second Partitioning Method

Δ = # discrepancies



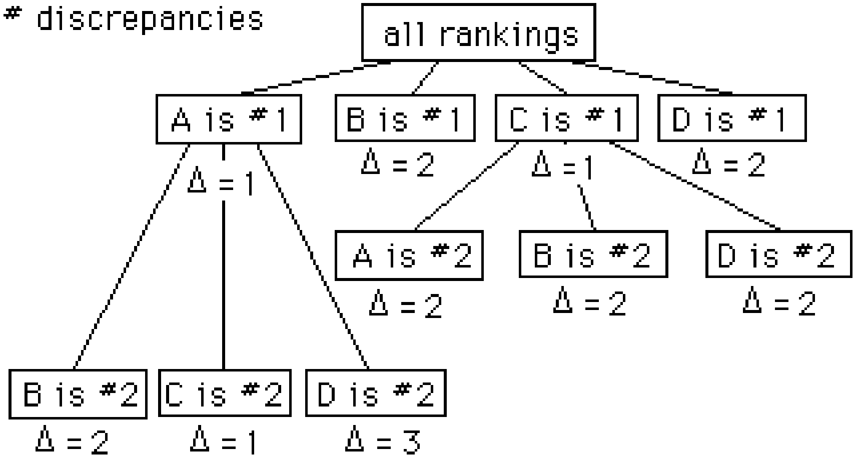
We will partition the most promising node, that with one discrepancy



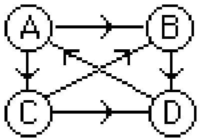
	A	B	C	D
A	–	1	1	0
B	0	–	0	1
C	0	1	–	1
D	1	0	0	–

Second Partitioning Method

Δ = # discrepancies



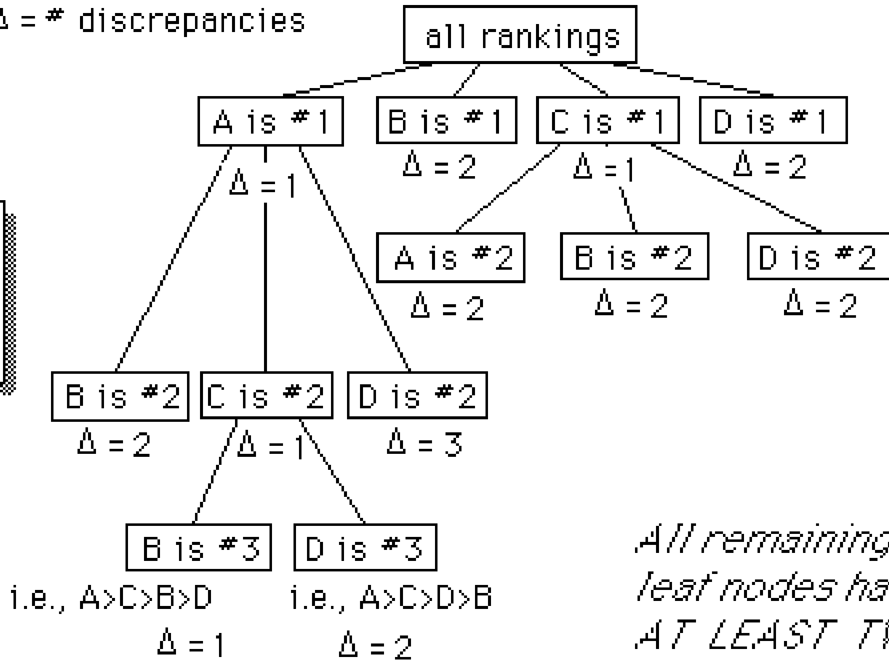
We will partition the most promising node, that with one discrepancy



	A	B	C	D
A	-	1	1	0
B	0	-	0	1
C	0	1	-	1
D	1	0	0	-

Second Partitioning Method

Δ = # discrepancies



All remaining leaf nodes have AT LEAST TWO discrepancies!