PAR, Inc. Golf Bags

Stochastic LP with Recourse

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PAR, Inc., a manufacturer of bags, must schedule its production for the next period.

PAR produces two models, and each model requires production time in each of four departments:

Production Time/Bag in each Department

Product	Cutting & Dyeing	Sewing	Finishing	Inspect & Packing
Standard	$^7/_{10}$ hr	½ hr	1 hr	$^1/_{10}$ hr
Deluxe	1 hr	$^5/_6$ hr	$^2/_3$ hr	¼ hr

The marketing department can sell as many bags as are produced, at a profit of **$10** per standard bag and **$9** per deluxe bag.

Given the times available in the four departments, an LP model can be formulated to plan the production:

$$\textit{Maximize} \quad 10X_1 + 9X_2$$

subject to:

$$\begin{cases} \frac{7}{10}X_1 + X_2 \le 630 \\ \frac{1}{2}X_1 + \frac{5}{6}X_2 \le 600 \\ X_1 + \frac{2}{3}X_2 \le 708 \\ \frac{1}{10}X_1 + \frac{1}{4}X_2 \le 136 \\ X_1 \ge 0, X_2 \ge 0 \end{cases}$$

However, the company has submitted bids on two government contracts which would, if successful, reduce the times available in the four production departments.

Production hours required to fulfill contracts:

Contract	Probability		C&D	SEW	FIN	I&P
#1	50%		50	40	80	10
#2	40%		30	50	70	15

Production for the golf bags must be done *before* learning which contracts have been won.

After learning the outcome of the bidding process, some (more costly) ***recourses*** are available:

- Adding *production* of standard golf bags
- Adding *overtime* in any of the four production departments

For each of the four scenarios, we compute the available time in the four production departments:

SCENARIO

Department	#0	#1	#2	#3
C&D	630	580	600	550
SEW	600	560	550	510
FIN	708	628	638	558
I&P	135	125	120	110

AVAILABLE HOURS

Recourses

- Scheduling **overtime** in
  - C&D at $5/hr
  - SEW at $6/hr
  - FIN at $8/hr *(but only up to 100 hrs)*
  - I&P at $4/hr
- Schedule additional **production** of standard bags, at a reduced profit of $8/bag

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2-stage Stochastic LP: **PAR Golf Bags**

K= # scenarios = 4
First-stage data:
A,B=
 1 1 ¯1 0 i.e., $X_1 + X_2 \geq 0$ *(added because APL code assumes non-empty constraint)*
Cost= ¯10 ¯9 0 *(negated because APL code assumes minimization)*

Second-stage data *(Only the RHS vector h varies by scenario)*

Scenario # 1, with probability 0.3
Cost= ¯8 5 6 8 4 0 0 0 0
T,W,h=

```
 0.7 1        0 |  0.7 ¯1  0  0  0 1 0 0 0 |  630
 0.5 0.833333 0 |  0.5  0 ¯1  0  0 0 1 0 0 |  600
 1   0.666667 0 |  1    0  0 ¯1  0 0 0 1 0 |  708
 0.1 0.25     0 |  0.1  0  0  0 ¯1 0 0 0 1 |  135
```

```
Scenario # 2, with probability 0.3
Cost= ¯8 5 6 8 4 0 0 0 0
T,W,h=
 0.7 1        0 |  0.7 ¯1  0  0  0 1 0 0 0 |  580
 0.5 0.833333 0 |  0.5  0 ¯1  0  0 0 1 0 0 |  560
 1   0.666667 0 |  1    0  0 ¯1  0 0 0 1 0 |  628
 0.1 0.25     0 |  0.1  0  0  0 ¯1 0 0 0 1 |  125
------------------------------------------------------------

Scenario # 3, with probability 0.3
Cost= ¯8 5 6 8 4 0 0 0 0
T,W,h=
 0.7 1        0 |  0.7 ¯1  0  0  0 1 0 0 0 |  600
 0.5 0.833333 0 |  0.5  0 ¯1  0  0 0 1 0 0 |  550
 1   0.666667 0 |  1    0  0 ¯1  0 0 0 1 0 |  638
 0.1 0.25     0 |  0.1  0  0  0 ¯1 0 0 0 1 |  120
------------------------------------------------------------
Scenario # 4, with probability 0.1
Cost= ¯8 5 6 8 4 0 0 0 0
T,W,h=
 0.7 1        0 |  0.7 ¯1  0  0  0 1 0 0 0 |  550
 0.5 0.833333 0 |  0.5  0 ¯1  0  0 0 1 0 0 |  510
 1   0.666667 0 |  1    0  0 ¯1  0 0 0 1 0 |  558
 0.1 0.25     0 |  0.1  0  0  0 ¯1 0 0 0 1 |  110
------------------------------------------------------------
```

Certainty-Equivalent LP

i.e., replacing all random parameters by their expected values.

Tableau

b	z	X[1]	X[2]	X[3]	1	2	3	4	5	6	7	8	9
0	1	¯10	¯9	0	¯8	5	6	8	4	0	0	0	0
0	0	1	1	¯1	0	0	0	0	0	0	0	0	0
598	0	0.7	1	0	0.7	¯1	0	0	0	1	0	0	0
564	0	0.5	0.8333	0	0.5	0	¯1	0	0	0	1	0	0
648	0	1	0.6667	0	1	0	0	¯1	0	0	0	1	0
125	0	0.1	0.25	0	0.1	0	0	0	¯1	0	0	0	1

Solution of Certainty Equivalent Problem

Total cost: ¯ 7111.749374

Stage One Variables:

i	variable	value
1	Standard	467.4998308
2	Deluxe	270.7501185

Second Stage

i	variable	value
1	Add standard	0.00000000
2	OT C&D	0.00000000
3	OT Sew	0.00000000
4	OT Finish	0.00000000
5	OT I&P	0.00000000
6	slack C&D	0.00000000
7	slack Sew	104.62507615
8	slack Finish	0.00000000
9	slack I&P	10.56248731

What is the penalty if this solution is used? -- Value of Stochastic Solution (VSS)

Evaluate solution found when using expected values of parameters:

Evaluation of trial solution

i	X[i]	
1	467	Standard bags
2	270	Deluxe bags

Second stage costs:
(assuming recourse is chosen after the scenario is observed)

scenario k	cost	p[k]
1	¯39.59996355	0.3
2	21.28506480	0.3
3	6.48006480	0.3
4	9.63300720	0.1

First stage cost:	¯ 7100.00
Expected second stage cost:	56.88
Total:	¯ 7043.12

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Tableau of Deterministic Equivalent LP:

b	z	[1]	2]	3]	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7
0	1	¯10	¯9	0	¯2.4	1.5	1.8	2.4	1.2	0	0	0	0	¯2.4	1.5	1.8	2.4	1.2	0	0
0	0	1	1	¯1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
630	1	0	0	0	0.7	¯1	0	0	0	1	0	0	0	0	0	0	0	0	0	0
600	0	1	0	0	0.5	0	¯1	0	0	0	1	0	0	0	0	0	0	0	0	0
708	0	0	1	0	1	0	0	¯1	0	0	0	1	0	0	0	0	0	0	0	0
135	0	0	0	1	0.1	0	0	0	¯1	0	0	0	1	0	0	0	0	0	0	0
580	1	0	0	0	0	0	0	0	0	0	0	0	0	0.7	¯1	0	0	0	1	0
560	0	1	0	0	0	0	0	0	0	0	0	0	0	0.5	0	¯1	0	0	0	1
628	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	¯1	0	0	0
125	0	0	0	1	0	0	0	0	0	0	0	0	0	0.1	0	0	0	¯1	0	0
600	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
550	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
638	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
120	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
550	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
510	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
558	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
110	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Continued….

(LP Tableau, continued)

8	9	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
0	0	¯2.4	1.5	1.8	2.4	1.2	0	0	0	0	¯0.8	0.5	0.6	0.8	0.4	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0.7	¯1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0.5	0	¯1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
0	0	1	0	0	¯1	0	0	0	1	0	0	0	0	0	0	0	0	0	0
0	0	0.1	0	0	0	¯1	0	0	0	1	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0.7	¯1	0	0	0	1	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0.5	0	¯1	0	0	0	1	0	0
0	0	0	0	0	0	0	0	0	0	0	1	0	0	¯1	0	0	0	1	0
0	0	0	0	0	0	0	0	0	0	0	0.1	0	0	0	¯1	0	0	0	1

Optimal Solution

Found by solving deterministic equivalent problem directly,
(without decomposition)

Total cost: ¯ 7046.199413

Stage One Variables:

i	variable	value
1	Standard	497.1426996
2	Deluxe	252.0001103
3	surplus 1	749.1428099

Second Stage

Scenario #1

i	variable	value
1	Add standard	42.85714286
7	slack Sew	120.00007088
9	slack I&P	17.99998819

Scenario #2

i	variable	value
2	OT C&D	20.00000000
4	OT Finish	37.14285714
7	slack Sew	101.42864230
9	slack I&P	12.28570247

Scenario #3

i	variable	value
4	OT Finish	27.142857143
7	slack Sew	91.428642304
9	slack I&P	7.285702473

Scenario #4

i	variable	value
2	OT C&D	50.000000000
4	OT Finish	107.142857143
5	OT I&P	2.714297527
7	slack Sew	51.428642304

Value of Stochastic Solution *(VSS)*

Let x^* = optimal solution of the *stochastic* problem

& $\bar{x}$ = optimal solution of the *deterministic* problem with times available equal to their *expected* values.

VSS = { E_ξ[profit (x*)] } − { E_ξ [profit (x*)]

= 7046.20 − 7043.12 = **3.08**

(only 0.04% of optimal profit)

(This suggests that, for this particular problem, there is not much value in considering the stochastic nature of the available times!)

Assuming "Perfect Information"

Here we compute the optimal solution for each scenario, *assuming that at stage one the second stage outcome is known in advance.*

That is, we solve a problem for each scenario, assuming *both* stage #1 and stage #2 variables may be chosen.

Solution for scenario #1

Won neither contract

Optimal cost: 7667.999417

Stage One Variables:

i	X[i]	
1	540.00	***Standard***
2	252.00	***Deluxe***

Second-stage: nonzero variables

i	Y[i]	
7	120.00	***slack Finish***
9	18.00	***slack I&P***

Solution for scenario #2

Won contract #1

Optimal cost: 6894.249391

Stage One Variables:

i	X[i]	
1	452.50	***Standard***
2	263.25	***Deluxe***

Second-stage: nonzero variables

i	Y[i]	
7	114.38	***slack Finish***
9	13.94	***slack I&P***

Solution for scenario #3

Won contract #2

Optimal cost: 7051.124335

Stage One Variables:

i	X[i]	
1	446.25	***Standard***
2	287.63	***Deluxe***

Second-stage: nonzero variables

i	Y[i]	
7	87.19	***slack Finish***
9	3.47	***slack I&P***

Solution for scenario #4

Won both contracts

Optimal cost: 6274.999253

Stage One Variables:

i	X[i]	
1	358.75	***Standard***
2	298.88	***Deluxe***

Second-stage: nonzero variables

i	Y[i]	
5	0.59	***slack Finish***
7	81.56	***slack I&P***

Expected cost with perfect information:

$$0.3\times(7667.99)+0.3\times(6894.25)+0.3\times(7051.12)+0.1\times(6274.99)$$
$$=-7111.51$$

Value of Perfect Information (VPI):

VWPI – VWOI = 7111.51 – 7046.19 = 65.32

(= 0.9% of VWOI)

That is, it would be worth only $65.32 in additional profits if they were able to learn the outcome of the bidding process before scheduling production of the golf bags!