Stochastic Linear Programming

with Recourse

Dept of Industrial Engineering
The University of Iowa

Consider the **deterministic-equivalent LP** derived from the 2-stage stochastic LP:

$$Z = \min cx + \sum_{k=1}^{K} p_k q_k y_k \tag{0.1}$$

subject to

$$T_k x + W y_k = h_k, k = 1, \ldots K; \tag{0.2}$$

$$x \in X \tag{0.3}$$

$$y_k \geq 0, k = 1, \ldots K \ldots \tag{0.4}$$

where, for example, the feasible set of first-stage decisions is defined by

$$X = \left\{ x \in R^n : Ax = b, x \geq 0 \right\} \tag{0.5}$$

Note that in general, the parameters q, T, h (and sometimes W) may vary by scenario!

Here k indexes the finitely-many possible ***realizations*** of a random vector ξ, with p_k the probability of realization k.

The set of **first-stage decision variables** x are to be selected *before* ξ is observed.

Then the set of **second-stage decision variables** y_k are to be selected once x has been selected and the k^{th} realization of ξ is observed.

Note that in general, the coefficient matrices T and W, the right-hand-side vector h, and the second-stage cost vector q are all ***random***.

We assume here that for any choice of x and realization ξ, the constraints (0.2) are feasible in y, a condition known as ***complete*** recourse. (This may require the introduction of *artificial* variables with large costs.)

The objective is to minimize the **expected total costs** of first and second stages

$$\text{Minimize } cx + \sum_{k=1}^{K} p_k Q_k(x) \qquad (0.6)$$

$$\text{subject to } x \in X$$

where the cost of the second stage is

$$Q_k(x) = \text{Minimum } \{ q_k y : W_k y = h_k - T_k x,\ y \geq 0 \} \qquad (0.7)$$

The function $Q_k(x)$ is nonlinear and costly to evaluate for any x, but has some nice properties (e.g., convexity and continuity).