Giapetto Toys

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Giapetto, Inc. manufactures and sells two wooden products:

- sets of toy soldiers
- toy trains,

using only two resources:

- lumber
- labor.

The toys can be made from either

- Grade **A** lumber, *or* (allowing for scrap)
- Grade **B** lumber:

Resource \ product	Soldier Set	Train
Grade A Lumber	3 board feet	5 board feet
Grade B Lumber	4 board feet	8 board feet
Labor	2 hours	4 hours

90,000 hours of labor will be available for production.

A shipment of 150,000 board feet of lumber will be received before production begins.

The quality of the lumber is uncertain, however, and will not be determined until *after* the production is scheduled.

Based upon past experience with the supplier, the following cases with their probabilities have been identified:

Case:	1	2	3
Probability	25%	50%	25%
Grade A	125,000 bd ft	100,000 bd ft	75,000 bd ft
Grade B	25,000 bd ft	50,000 bd ft	75,000 bd ft

Demand for the products is also uncertain, with two cases having been identified:

Case:	1	2
Probability	40%	60%
Sets of soldiers	40,000	50,000
Toy trains	60,000	80,000

The revenue from sale of toy trains is $50 and that from a set of toy soldiers is $40.

Production quantities of the two products must be fixed *before* the lumber arrives and *before* the levels of demand are known.

After the lumber quality has been determined and the demand experienced, the company has the following *recourses* available:

- Buy sets of toy soldiers from another supplier at $35 each
- Buy trains at $45 each from another supplier
- Schedule overtime at an additional cost of $10/hour

The six scenarios which can occur, with their probabilities.

Scenario #	*Probability*	*Lumber quality*	*Demand*
1			
2			
3			
4			
5			
6			

b. Compute the expected value of the random elements of the problem, and solve the LP to determine the optimal production quantities if the expected values were to occur:

# sets of toy soldiers	*# wooden trains*

What is the optimal value of the LP? $____________

c. Using the production quantities found in (b), compute the optimal *recourses* for each scenario, the profit for each scenario.

Scenario	# soldier sets purchased	# trains purchased	overtime labor	profit
1				
2				
3				
4				
5				
6				

Expected profit: $___________

d. Find the production quantities which maximize the expected profits by solving the "deterministic equivalent" LP.

- What are the production quantities?

# sets of toy soldiers	*# wooden trains*

- What are the dimensions of the LP tableau? _______ × ________
- What is the maximum expected profit? $_________
- What is the recourse if the lumber is of the lowest quality (case #1) and the demand is the higher estimate (case #2)?

Scenario	*# soldier sets purchased*	*# trains purchased*

- What is the *value of the stochastic solution (VSS)*, i.e., the difference in expected profit when using the stochastic LP solution compared to the LP using the expected values? $________

Model definition:

Decision variables

- **First Stage:**

  X_1 = # of sets of soldiers to be produced

  X_2 = # toy trains to be produced

- **Second Stage (Recourse):**

  Y_{1A} & Y_{1B} = # of sets of soldiers to be produced from Grade A and Grade B lumber, respectively

  Y_{2A} & Y_{2B} = # of toy trains to be produced from Grade A and Grade B lumber, respectively

  Z_1 & Z_2 = # of sets of soldiers and trains, respectively, to be purchased from outside source

  S_1 & S_2 = # of sets of soldiers and trains, respectively, sold

  T = # hours of overtime used

Second Stage Optimization Problem:

After X has selected in the first stage, and we have observed the random values of

- A & B (the quantities of Grades A & B lumber, respectively), and
- D_1 and D_2 (the demands for toy soldiers and trains, respectively).

Find the maximum revenue minus costs:

$$Q(X) = Maximum \quad 40S_1 + 50S_2 - 35Z_1 - 45Z_2 - 10T$$

subject to

Production schedule is fulfilled: $\begin{cases} Y_{1A} + Y_{1B} = X_1 \\ Y_{2A} + Y_{2B} = X_2 \end{cases}$

Lumber resource constraints:
where A & B are the random available quantities of Grades A & B lumber, in board feet.

Labor resource constraint: $2X_1 + 4X_2 - T = 90000$

Sales consist of produced & purchased products: $\begin{cases} S_1 = Y_{1A} + Y_{1B} + Z_1 \\ S_2 = Y_{2A} + Y_{2B} + Z_2 \end{cases}$

Sales limited by demand: $\begin{cases} S_1 \le D_1 \\ S_2 \le D_2 \end{cases}$

where D_1 & D_2 are the random demands for the two products.

nonnegativity: $Y_{1A}, Y_{1B}, Y_{2A}, Y_{2B}, S_1, S_2, Z_1, Z_2, \& \ T \geq 0$

First-stage Optimization Problem:

$$\underset{X \geq 0}{Maximize} \quad \sum_{k \in K} p_k Q_k(X)$$

where K is the set of scenarios, and

p_k the probability of scenario #k.

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First-stage data:

A,B=

1 1 > 0

i	variable	cost
1	X[1]	0
2	X[2]	0

Objective: Minimize

Second-stage data

K= # scenarios = 6

The following data vary by scenario: h

Costs:

i	var.	q
1	Y1A	0
2	Y1B	0
3	Y2A	0
4	Y2B	0
5	Z1	35
6	Z2	45
7	S1	¯40
8	S2	¯50
9	T	10

Technology matrix T (coefficients of X in 2nd stage) =

¯1	0
0	¯1
0	0
0	0
2	4
0	0
0	0
0	0
0	0

Technology matrix W (coefficients of Y in 2nd stage) =

1	1	0	0	0	0	0	0	0
0	0	1	1	0	0	0	0	0
3	0	5	0	0	0	0	0	0
0	4	0	8	0	0	0	0	0
0	0	0	0	0	0	0	0	¯1
1	1	0	0	1	0	¯1	0	0
0	0	1	1	0	1	0	¯1	0
0	0	0	0	0	0	1	0	0
0	0	0	0	0	0	0	1	0

Right-hand-sides in second stage =

k	p[k]	1	2	3	4	5	6	7	8	9

1	0.1	0	0	125000	25000	90000	0	0	40000	60000
2	0.2	0	0	100000	50000	90000	0	0	40000	60000
3	0.1	0	0	75000	75000	90000	0	0	40000	60000
4	0.15	0	0	125000	25000	90000	0	0	50000	80000
5	0.3	0	0	100000	50000	90000	0	0	50000	80000
6	0.15	0	0	75000	75000	90000	0	0	50000	80000

Note: h varies by scenario

Optimal Solution
(Found by solving deterministic equivalent
problem directly, without decomposition)
Total objective function: ¯2091250

Stage One: nonzero variables:

i	variable	value
1	X[1]	40000
2	X[2]	2250
3	surplus_1	42250

Second Stage: nonzero variables

Scenario #1

i	variable	value
1	Y1A	38250
2	Y1B	1750
4	Y2B	2250
6	Z2	57750
7	S1	40000
8	S2	60000
10	slack_3	10250
12	slack_5	1000

Scenario #2

i	variable	value
1	Y1A	32000
2	Y1B	8000
4	Y2B	2250
6	Z2	57750
7	S1	40000

8	S2	60000
10	slack_3	4000
12	slack_5	1000

Scenario #3

i	variable	value
1	Y1A	21250
2	Y1B	18750
3	Y2A	2250
6	Z2	57750
7	S1	40000
8	S2	60000
12	slack_5	1000

Scenario #4

i	variable	value
1	Y1A	38250
2	Y1B	1750
4	Y2B	2250
5	Z1	10000
6	Z2	77750
7	S1	50000
8	S2	80000
10	slack_3	10250
12	slack_5	1000

Scenario #5

i	variable	value
1	Y1A	32000
2	Y1B	8000
4	Y2B	2250
5	Z1	10000
6	Z2	77750
7	S1	50000
8	S2	80000
10	slack_3	4000
12	slack_5	1000

Scenario #6

i	variable	value
1	Y1A	21250
2	Y1B	18750
3	Y2A	2250
5	Z1	10000
6	Z2	77750
7	S1	50000
8	S2	80000
12	slack_5	1000

Solution with Perfect Information

Solution for scenario #1

Optimal cost: ¯2023750

Stage One: nonzero variables:

i	value	name

1	40000.00	X[1]
2	4750.00	X[2]
3	44750.00	surplus_1

Second-stage: nonzero variables

i	value	name
1	33750.00	Y1A
2	6250.00	Y1B
3	4750.00	Y2A
6	55250.00	Z2
7	40000.00	S1
8	60000.00	S2
9	9000.00	T

Solution for scenario #2

Optimal cost: ¯2017500

Stage One: nonzero variables:

i	value	name
1	40000.00	X[1]
2	3500.00	X[2]
3	43500.00	surplus_1

Second-stage: nonzero variables

i	value	name
1	27500.00	Y1A
2	12500.00	Y1B
3	3500.00	Y2A
6	56500.00	Z2
7	40000.00	S1
8	60000.00	S2
9	4000.00	T

Solution for scenario #3

Optimal cost: ¯2001250

Stage One: nonzero variables:

i	value	name
1	40000.00	X[1]
2	2250.00	X[2]
3	42250.00	surplus_1

Second-stage: nonzero variables

i	value	name
1	21250.00	Y1A
2	18750.00	Y1B
3	2250.00	Y2A
6	57750.00	Z2
7	40000.00	S1
8	60000.00	S2
12	1000.00	slack_5

Solution for scenario #4

Optimal cost: ¯2268750

Stage One: nonzero variables:

i	value	name

1	47916.67	X[1]
3	47916.67	surplus_1

Second-stage: nonzero variables

i	value	name
1	41666.67	Y1A
2	6250.00	Y1B
5	2083.33	Z1
6	80000.00	Z2
7	50000.00	S1
8	80000.00	S2
9	5833.33	T

Solution for scenario #5

Optimal cost: ¯2237500

Stage One: nonzero variables:

i	value	name
1	45833.33	X[1]
3	45833.33	surplus_1

Second-stage: nonzero variables

i	value	name
1	33333.33	Y1A
2	12500.00	Y1B
5	4166.67	Z1
6	80000.00	Z2
7	50000.00	S1
8	80000.00	S2
9	1666.67	T

Solution for scenario #6

Optimal cost: ¯2181250

Stage One: nonzero variables:

i	value	name
1	43750.00	X[1]
3	43750.00	surplus_1

Second-stage: nonzero variables

i	value	name
1	25000.00	Y1A
2	18750.00	Y1B
5	6250.00	Z1
6	80000.00	Z2
7	50000.00	S1
8	80000.00	S2
12	2500.00	slack_5

Expected cost with perfect information: ¯2144750

Certainty-Equivalent Tableau

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b	z	1	2	3	1	2	3	4	5	6	7	8	9	0	1	2	3	4
0	1	0	0	0	0	0	0	0	35	45	¯40	¯50	10	0	0	0	0	0
0	0	1	1	¯1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	¯1	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	¯1	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0
100000	0	0	0	0	3	0	5	0	0	0	0	0	0	1	0	0	0	0
50000	0	0	0	0	0	4	0	8	0	0	0	0	0	0	1	0	0	0
90000	0	2	4	0	0	0	0	0	0	0	0	0	¯1	0	0	1	0	0
0	0	0	0	0	1	1	0	0	1	0	¯1	0	0	0	0	0	0	0
0	0	0	0	0	0	0	1	1	0	1	0	¯1	0	0	0	0	0	0
46000	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	0
72000	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1

Optimal Solution
Found by solving certainty equivalent problem,
i.e., replacing all random parameters by their expected values.

Total objective function: ¯2177500
Stage One: nonzero variables:

i	variable	value
1	X[1]	45833.33333
3	surplus_1	45833.33333

Second Stage: nonzero variables

i	variable	value

1	Y1A	33333.3333333
2	Y1B	12500.0000000
5	Z1	166.6666667
6	Z2	72000.0000000
7	S1	46000.0000000
8	S2	72000.0000000
9	T	1666.6666667