

Signomial GP Algorithm

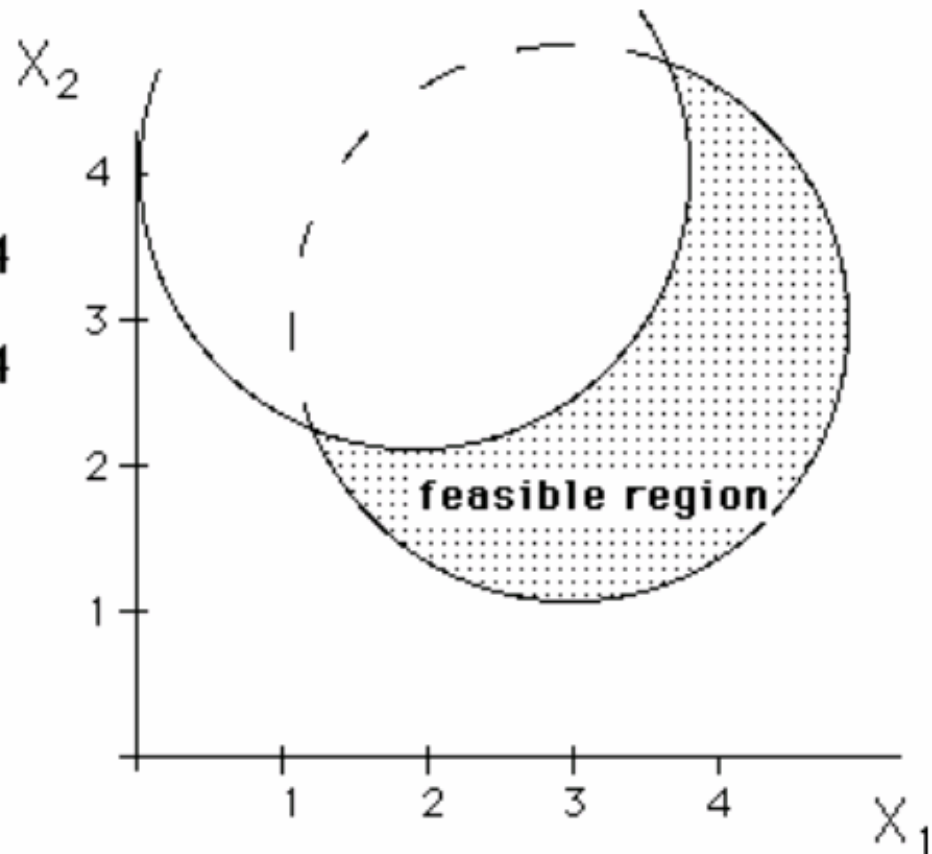
successive posynomial approximations

Minimize x_1

subject to

$$(x_1 - 2)^2 + (x_2 - 4)^2 \geq 4$$

$$(x_1 - 3)^2 + (x_2 - 3)^2 \leq 4$$



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Dept of Mechanical & Industrial Engineering
The University of Iowa

Minimize x_1
subject to

$$\frac{x_1}{4} + \frac{x_2}{2} - \frac{x_1^2}{16} - \frac{x_2^2}{16} \leq 1$$

$$\frac{x_1^2 x_2^{-1}}{6} + \frac{x_2}{6} + \frac{7x_2^{-1}}{3} - x_1 x_2^{-1} \leq 1$$

Coefficients and exponent matrix:

| t | p | Ct | expon | |
|---|---|---------|-------|----|
| 1 | 1 | 1 | 1 | 0 |
| 2 | 2 | 0.25 | 1 | 0 |
| 3 | 2 | 0.5 | 0 | 1 |
| 4 | 2 | -0.0625 | 2 | 0 |
| 5 | 2 | -0.0625 | 0 | 2 |
| 6 | 3 | 0.16667 | 2 | -1 |
| 7 | 3 | 0.16667 | 0 | 1 |
| 8 | 3 | 2.33333 | 0 | -1 |
| 9 | 3 | -1 | 1 | -1 |

t = term number, p = polynomial, Ct = coefficient

Bounds on variables

| # | var | LB | UB |
|---|--------|-----|----|
| 1 | X[1] | 0.1 | 10 |
| 2 | X[2] | 0.1 | 10 |

**reformulated
as signomial GP**

Initial point at which condensation of signomial is performed: (4,4)

Major Iteration # 1

Signomial Problem at x = 4 4
Values of terms (Ut)

| t | p | Ut |
|---|---|----------|
| 1 | 1 | 4 |
| 2 | 2 | 1 |
| 3 | 2 | 2 |
| 4 | 2 | -1 |
| 5 | 2 | -1 |
| 6 | 3 | 0.666666 |
| 7 | 3 | 0.666664 |
| 8 | 3 | 0.583333 |
| 9 | 3 | -1 |

Objective function = 4

| Posy | Value | Infeas |
|------|----------|--------|
| 2 | 1.000000 | 0 |
| 3 | 0.916664 | 0 |

Weights of negative terms, used for condensation:

| poly | term | value |
|------|------|----------|
| 2 | 3 | 0.333333 |
| 2 | 4 | 0.333333 |
| 3 | 4 | 0.500000 |

Condensation of Signomial GP at x= 4 4

Number of variables: 1

Number of posynomials: 7

Total number of terms: 10

Degrees of difficulty: 8

Terms per posynomial: 1 2 3 1 1 1 1

(includes bounds on variables to ensure dual feasibility)

Coefficients and exponent matrix:

| t | p | Ct | exponents | |
|----|---|-----------|-----------|-----------|
| 1 | 1 | 1 | 1 | 0 |
| 2 | 2 | 0.529134 | 0.333333 | -0.666667 |
| 3 | 2 | 1.05827 | -0.666667 | 0.333333 |
| 4 | 3 | 0.0833333 | 1.5 | -0.5 |
| 5 | 3 | 0.0833333 | -0.5 | 1.5 |
| 6 | 3 | 1.16667 | -0.5 | -0.5 |
| 7 | 4 | 0.1 | -1 | 0 |
| 8 | 5 | 0.1 | 0 | -1 |
| 9 | 6 | 0.1 | 1 | 0 |
| 10 | 7 | 0.1 | 0 | 1 |

t = term number, p = posynomial, Ct = coefficient

Posynomial GP via Generalized LP

| <u>i</u> | <u>X[i]</u> |
|----------|-------------|
| 1 | 3.99226 |
| 2 | 4.11993 |

Weights of terms:

posy#1: 1
 posy#2: 0.326375 0.673625
 posy#3: 0.339746 0.361822 0.298432
 posy#4: 1
 posy#5: 1
 posy#6: 1
 posy#7: 1

Posynomial objective functions:

Primal: 3.99226 Dual: 3.99226
 Duality Gap: 0 = 0 percent

Posynomial Constraints:

| | <u>Posy</u> | <u>Value</u> | <u>Infeas</u> | <u>Lambda</u> |
|--|-------------|--------------|---------------|---------------|
| | 2 | 1.0007557 | 0.000755699 | 3 |
| | 3 | 0.9639338 | 0.000000000 | 0 |
| | 4 | 0.0250485 | 0.000000000 | 0 |
| | 5 | 0.0242723 | 0.000000000 | 0 |
| | 6 | 0.3992259 | 0.000000000 | 0 |
| | 7 | 0.4119926 | 0.000000000 | 0 |

Condensation of Signomial GP at $x=$ **3.99226 4.11993**

Number of variables: 1

Number of posynomials: 7

Total number of terms: 10

Degrees of difficulty: 8

Terms per posynomial: 1 2 3 1 1 1 1

(includes bounds on variables to ensure dual feasibility)

Coefficients and exponent matrix:

| t | p | Ct | exponents | |
|----|---|-----------|-----------|-----------|
| 1 | 1 | 1 | 1 | 0 |
| 2 | 2 | 0.538556 | 0.348293 | -0.694055 |
| 3 | 2 | 1.07711 | -0.651707 | 0.305945 |
| 4 | 3 | 0.0833436 | 1.50787 | -0.507869 |
| 5 | 3 | 0.0833433 | -0.492131 | 1.49213 |
| 6 | 3 | 1.16681 | -0.492131 | -0.507869 |
| 7 | 4 | 0.1 | -1 | 0 |
| 8 | 5 | 0.1 | 0 | -1 |
| 9 | 6 | 0.1 | 1 | 0 |
| 10 | 7 | 0.1 | 0 | 1 |

Posynomial GP via Generalized LP

| <u>i</u> | <u>X[i]</u> |
|----------|-------------|
| 1 | 3.98431 |
| 2 | 4.78171 |

Weights of terms:

posy#1: 1
 posy#2: 0.294095 0.705905
 posy#3: 0.301004 0.43354 0.265456
 posy#4: 1
 posy#5: 1
 posy#6: 1
 posy#7: 1

Posynomial objective functions:

Primal: 3.98431 Dual: 3.98431
 Duality Gap: 0 = 0 percent

Posynomial Constraints:

| | <u>Posy</u> | <u>Value</u> | <u>Infeas</u> | <u>Lambda</u> |
|--|-------------|--------------|---------------|---------------|
| | 2 | 1.0004022 | 0.000402215 | 2.90242911 |
| | 3 | 1.0055781 | 0.005578142 | 0.00707249 |
| | 4 | 0.0250984 | 0.000000000 | 0.000000000 |
| | 5 | 0.0209130 | 0.000000000 | 0.000000000 |
| | 6 | 0.3984314 | 0.000000000 | 0.000000000 |
| | 7 | 0.4781705 | 0.000000000 | 0.000000000 |

Condensation of Signomial GP at x= **3.98431 4.78171**

Number of variables: 1

Number of posynomials: 7

Total number of terms: 10

Degrees of difficulty: 8

Terms per posynomial: 1 2 3 1 1 1 1

(includes bounds on variables to ensure dual feasibility)

Coefficients and exponent matrix:

| t | p | Ct | exponents | |
|----|---|-----------|-----------|-----------|
| 1 | 1 | 1 | 1 | 0 |
| 2 | 2 | 0.602138 | 0.419989 | -0.835401 |
| 3 | 2 | 1.20428 | -0.580011 | 0.164599 |
| 4 | 3 | 0.0836793 | 1.54548 | -0.545482 |
| 5 | 3 | 0.083679 | -0.454518 | 1.45452 |
| 6 | 3 | 1.17151 | -0.454518 | -0.545482 |
| 7 | 4 | 0.1 | -1 | 0 |
| 8 | 5 | 0.1 | 0 | -1 |
| 9 | 6 | 0.1 | 1 | 0 |
| 10 | 7 | 0.1 | 0 | 1 |

t = term number, p = posynomial, Ct = coefficient

Posynomial GP via Generalized LP

| <u>i</u> | <u>X[i]</u> |
|----------|-------------|
| 1 | 3.8346 |
| 2 | 4.8188 |

Weights of terms:

posy#1: 1
 posy#2: 0.284631 0.715369
 posy#3: 0.283181 0.447199 0.26962
 posy#4: 1
 posy#5: 1
 posy#6: 1
 posy#7: 1

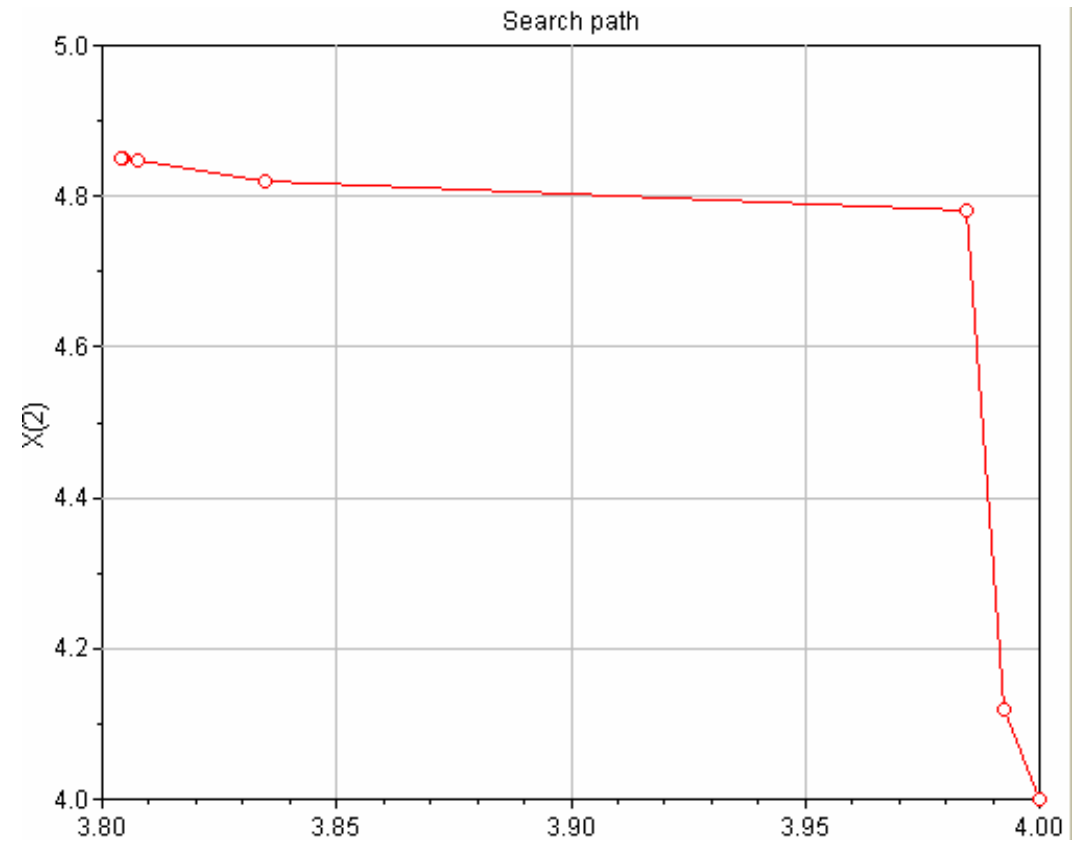
Posynomial objective functions:

Primal: 3.8346 Dual: 3.8346
 Duality Gap: 0 = 0 percent

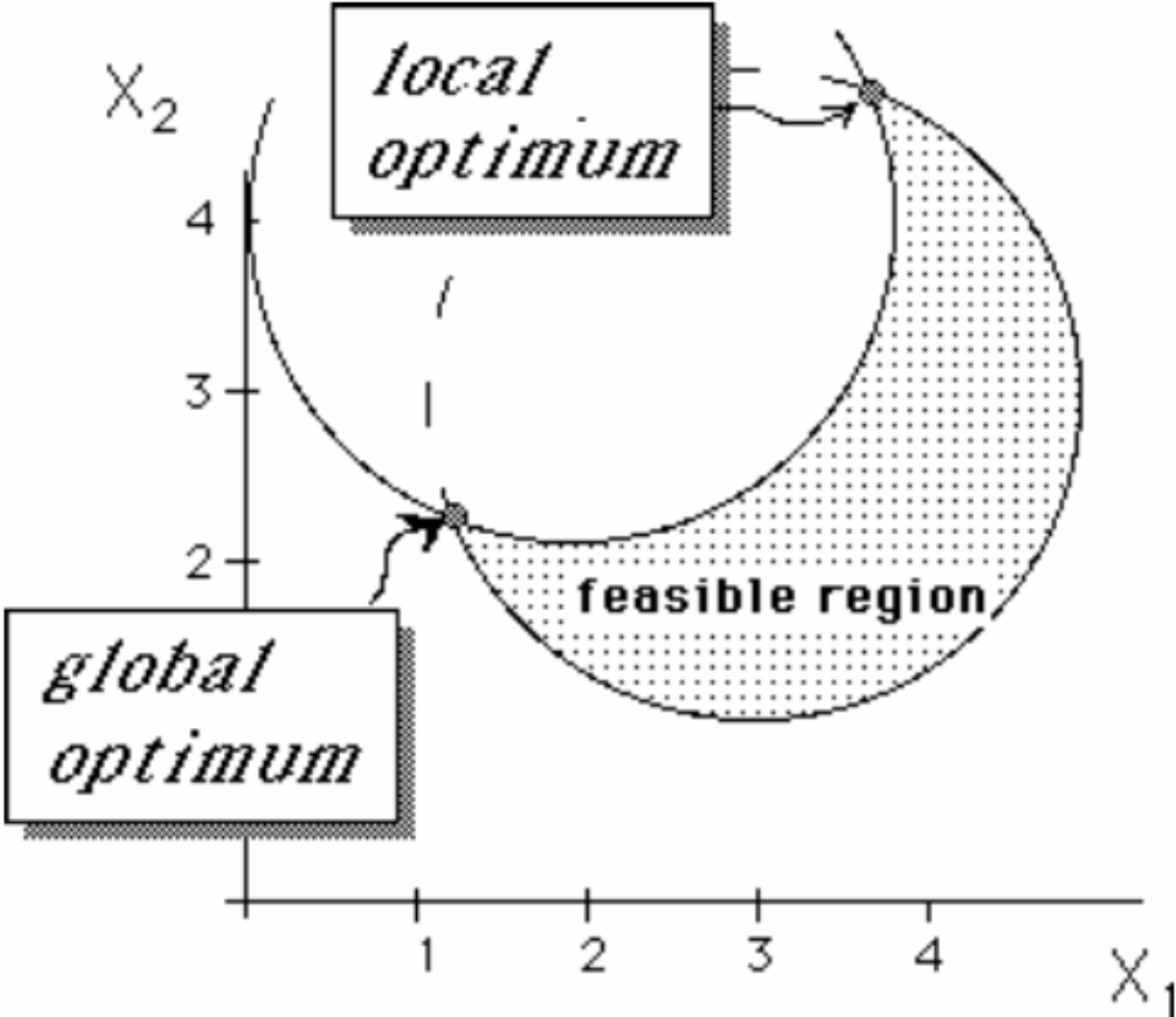
Posynomial Constraints:

| <u>Posy</u> | <u>Value</u> | <u>Infeasibility</u> | <u>Lambda</u> |
|-------------|--------------|----------------------|---------------|
| 2 | 1.0000965 | 0.0000965315 | 3.65706 |
| 3 | 1.0003488 | 0.0003487890 | 1.11362 |
| 4 | 0.0260783 | 0.0000000000 | 0.00000 |
| 5 | 0.0207521 | 0.0000000000 | 0.00000 |
| 6 | 0.3834600 | 0.0000000000 | 0.00000 |
| 7 | 0.4818800 | 0.0000000000 | 0.00000 |

The algorithm has converged
to a local (but not global)
optimum!



Minimize x_1



Condensation of Signomial GP at $x= 1 1$

Number of variables: 1
 Number of posynomials: 7
 Total number of terms: 10
 Degrees of difficulty: 8
 Terms per posynomial: 1 2 3 1 1 1 1
 (includes bounds on variables to ensure dual feasibility)

Coefficients and exponent matrix:

| t | p | Ct | exponents | |
|----|---|----------|-----------|----------|
| 1 | 1 | 1 | 1 | 0 |
| 2 | 2 | 0.22222 | 0.88889 | -0.11111 |
| 3 | 2 | 0.44444 | -0.11111 | 0.88889 |
| 4 | 3 | 0.083333 | 1.5 | -0.5 |
| 5 | 3 | 0.083333 | -0.5 | 1.5 |
| 6 | 3 | 1.1667 | -0.5 | -0.5 |
| 7 | 4 | 0.1 | -1 | 0 |
| 8 | 5 | 0.1 | 0 | -1 |
| 9 | 6 | 0.1 | 1 | 0 |
| 10 | 7 | 0.1 | 0 | 1 |

t = term number, p = posynomial, Ct = coefficient

Posynomial GP via Generalized LP

| <u>i</u> | <u>X[i]</u> |
|----------|-------------|
| 1 | 1.4491 |
| 2 | 1.7750 |

Weights of terms:

posy#1: 1
posy#2: 0.28987 0.71013
posy#3: 0.10908 0.16367 0.72726
posy#4: 1
posy#5: 1
posy#6: 1
posy#7: 1

Posynomial objective functions:

Primal: 1.4491 Dual: 1.4491
Duality Gap: 0 = 0 percent

Posynomial Constraints:

| <u>Posy</u> | <u>Value</u> | <u>Infeas</u> | <u>Lambda</u> |
|-------------|--------------|---------------|---------------|
| 2 | 1.000208 | 0.00020817 | 1.0676 |
| 3 | 1.000265 | 0.00026507 | 4.0165 |
| 4 | 0.069010 | 0.00000000 | 0.0000 |
| 5 | 0.056338 | 0.00000000 | 0.0000 |
| 6 | 0.144906 | 0.00000000 | 0.0000 |
| 7 | 0.177501 | 0.00000000 | 0.0000 |

Condensation of Signomial GP at $x=$ **1.4491 1.775**

Number of variables: 1

Number of posynomials: 7

Total number of terms: 10

Degrees of difficulty: 8

Terms per posynomial: 1 2 3 1 1 1 1

(includes bounds on variables to ensure dual feasibility)

Coefficients and exponent matrix:

| t | p | Ct | <u>exponents</u> | |
|----|---|----------|------------------|----------|
| 1 | 1 | 1 | 1 | 0 |
| 2 | 2 | 0.24012 | 0.80238 | -0.29653 |
| 3 | 2 | 0.48023 | -0.19762 | 0.70347 |
| 4 | 3 | 0.083761 | 1.5505 | -0.55055 |
| 5 | 3 | 0.083761 | -0.44945 | 1.4495 |
| 6 | 3 | 1.1727 | -0.44945 | -0.55055 |
| 7 | 4 | 0.1 | -1 | 0 |
| 8 | 5 | 0.1 | 0 | -1 |
| 9 | 6 | 0.1 | 1 | 0 |
| 10 | 7 | 0.1 | 0 | 1 |

t = term number, p = posynomial, Ct = coefficient

Posynomial GP via Generalized LP

| <u>i</u> | <u>X[i]</u> |
|----------|-------------|
| 1 | 1.2670 |
| 2 | 2.0835 |

Weights of terms:

posy#1: 1
posy#2: 0.23317 0.76683
posy#3: 0.080487 0.21763 0.70189
posy#4: 1
posy#5: 1
posy#6: 1
posy#7: 1

Posynomial objective functions:

Primal: 1.267 Dual: 1.267
Duality Gap: 0 = 0 percent

Posynomial Constraints:

| <u>Posy</u> | <u>Value</u> | <u>Infeas</u> | <u>Lambda</u> |
|-------------|--------------|---------------|---------------|
| 2 | 1.001607 | 0.0016069 | 0.98293 |
| 3 | 1.002754 | 0.0027541 | 3.20132 |
| 4 | 0.078924 | 0.0000000 | 0.00000 |
| 5 | 0.047997 | 0.0000000 | 0.00000 |
| 6 | 0.126705 | 0.0000000 | 0.00000 |
| 7 | 0.208348 | 0.0000000 | 0.00000 |

