Introduction to QUEUEING: M/G/1

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M/G/1

- *Arrival process is **M**emoryless, i.e., interarrival times have Exponential distribution with mean* $1/\lambda$
- *Single server*
- *Service times are independent, identically-distributed, but not necessarily exponential. Mean service time is* $1/\mu$ *with variance* σ^2
- *Queue capacity is infinite*

M/G/1

Steadystate Characteristics

A steadystate distribution exists if $\rho = \frac{\lambda}{\mu} < 1$

i.e., if service rate exceeds the arrival rate.

$\pi_0 = 1 - \rho$ = *probability that server is idle*

$1 - \pi_0 = \rho$ = *probability that server is busy*

i.e., utilization of server

There is no convenient formula for the probability of j customers in system when $j > 0$.

M/G/1

Steadystate Characteristics

$$L_q = \frac{\lambda^2 \sigma^2 + \rho^2}{2(1-\rho)}$$

average number of customers waiting

After calculating L_q, Little's Formula allows us to compute:

$$W_q = \frac{L_q}{\lambda}, \quad W = W_q + \frac{1}{\mu},$$

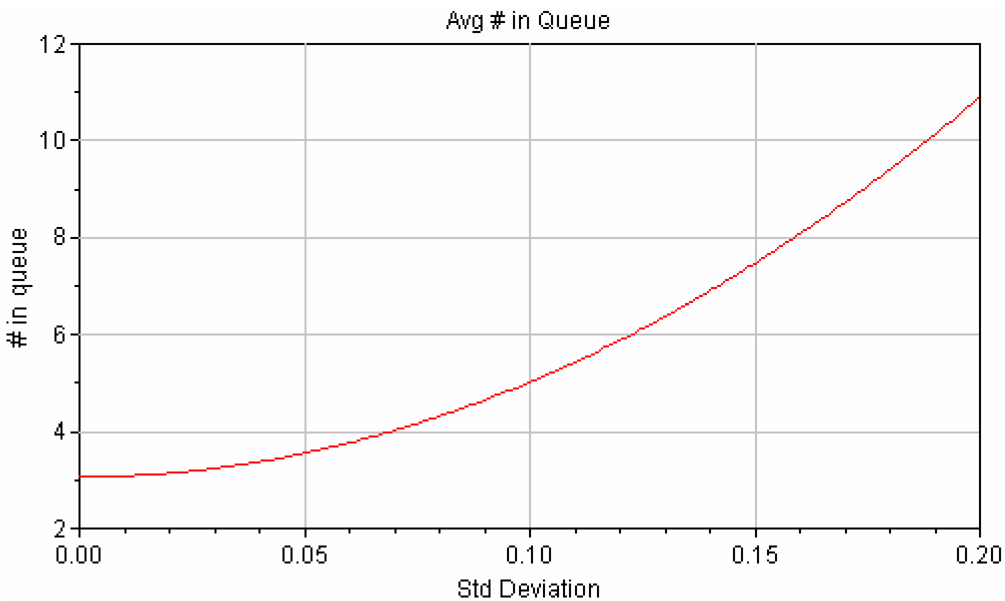
$$\& \quad L = \lambda W = L_q + \rho$$

For the M/M/1 queue, the standard deviation equals the mean service time, i.e., $\sigma = 1/\mu$

Using these formulae for the M/G/1 queueing system with $\sigma^2 = 1/\mu^2$ will give results consistent with the formulae for M/M/1.

Average number of customers waiting in the queue:

$$L_q = \frac{\lambda^2\sigma^2 + \rho^2}{2(1-\rho)}$$



What is the effect of variability in the service time on L_q?

Illustration:

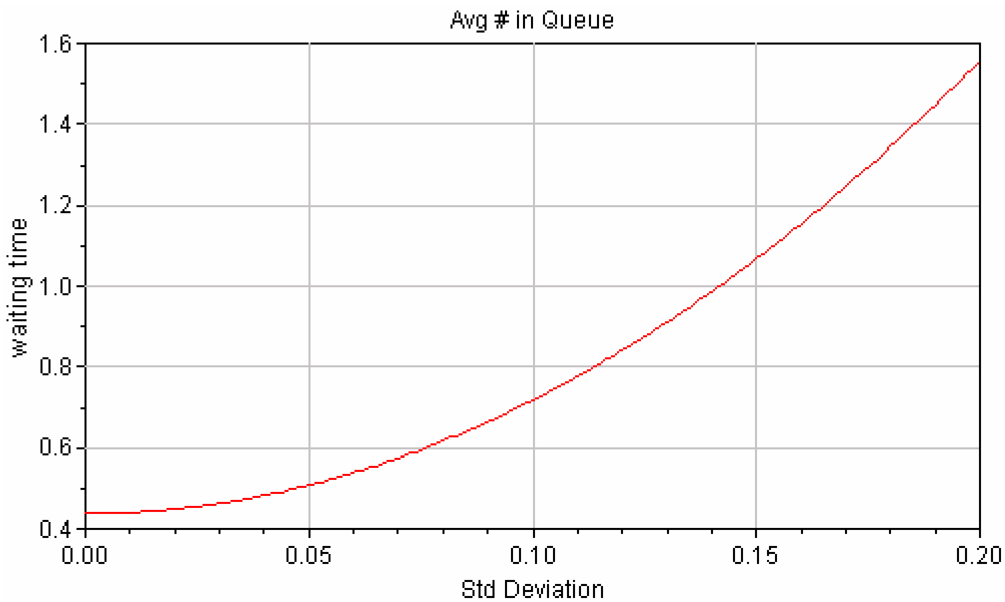
λ = 7/hr

μ = 8/hr

$\sigma \in$ [0, 0.2]

(For exponential distribution, σ=mean=$\frac{1}{\mu}$= 0.125 hr.)

Average waiting time = $W_q = L_q/\lambda$



What is the effect of variability in the service time on W_q?

Illustration:

$\lambda = 7/hr$

$\mu = 8/hr$

$\sigma \in [0, 0.2]$