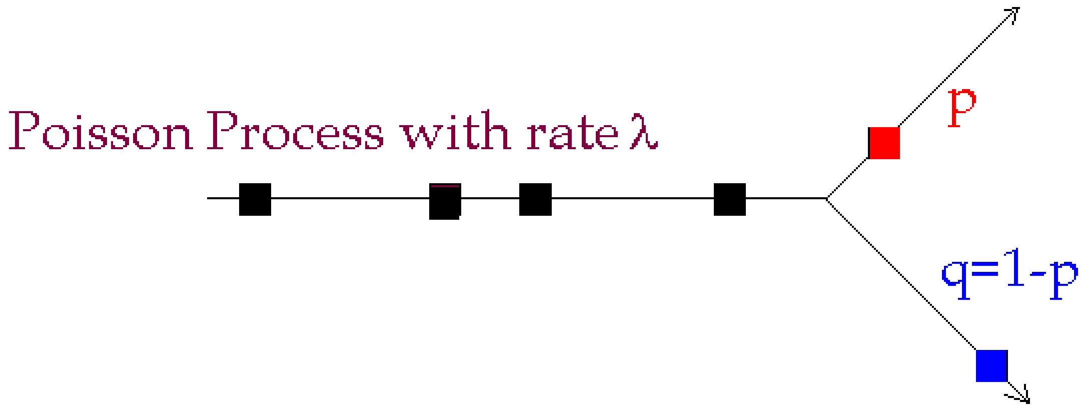
SPLITTING & MERGING OF POISSON PROCESSES



Consider a Poisson process with arrival rate λ and **"split"** it as follows:

each arrival is colored red with probability p, and

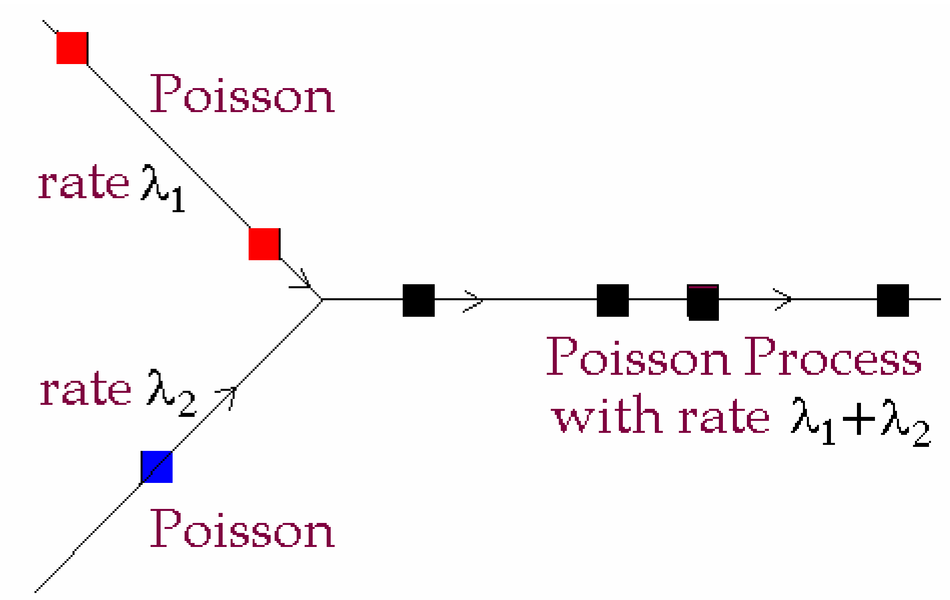
blue with probability q=1−p.

Then the process of red arrivals is a Poisson process with rate pλ, and

the process of blue arrivals is a Poisson process with rate qλ.

Example: *if the arrival of vehicles at an intersection is Poisson with rate 20/minute, and 30% of the vehicles are trucks, then the arrival of trucks is a Poisson process with rate 0.3×20/minute = 6/minute.*

Merging: Conversely, consider two Poisson processes with rates λ_1 and λ_2, and define a new process with an arrival whenever an arrival occurs in either process.



This new process is also Poisson, with arrival rate $\lambda_1 + \lambda_2$.

Example: the arrival of customers wishing to make a deposit at a bank teller window is Poisson with rate 9/hour, and the arrival of customers wishing to make a withdrawal is Poisson with rate 6/hour. The aggregate arrival of customers at this bank teller window is Poisson, with rate 15/hour.

MINIMUM OF EXPONENTIALLY-DISTRIBUTED RANDOM VARIABLES

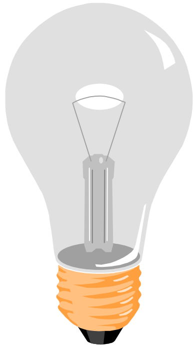
Suppose that $\mathbf{T_1}$ and $\mathbf{T_2}$ are independent exponentially-distributed random variables with parameters λ_1 and λ_2, respectively.

What is the distribution of the new random variable $\mathbf{T}$ defined as

$$\mathbf{T} = \min\{\mathbf{T}_1, \mathbf{T}_2\}?$$

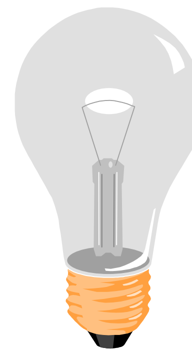
Example:

T_1 and T_2 are the lifetimes of two light bulbs, and T is the time at which the first failure occurs.



Think of $\mathbf{T}_1$ and $\mathbf{T}_2$ as the inter-arrival times of two Poisson processes, and *merge* them.

Then the time of the *next* arrival of the merged process is



$$\mathbf{T} = \text{minimum}\{\mathbf{T}_1, \mathbf{T}_2\}$$

As we have seen, therefore,

T has an *exponential* distribution with parameter $\lambda_1 + \lambda_2$.