"Passing the Buck"

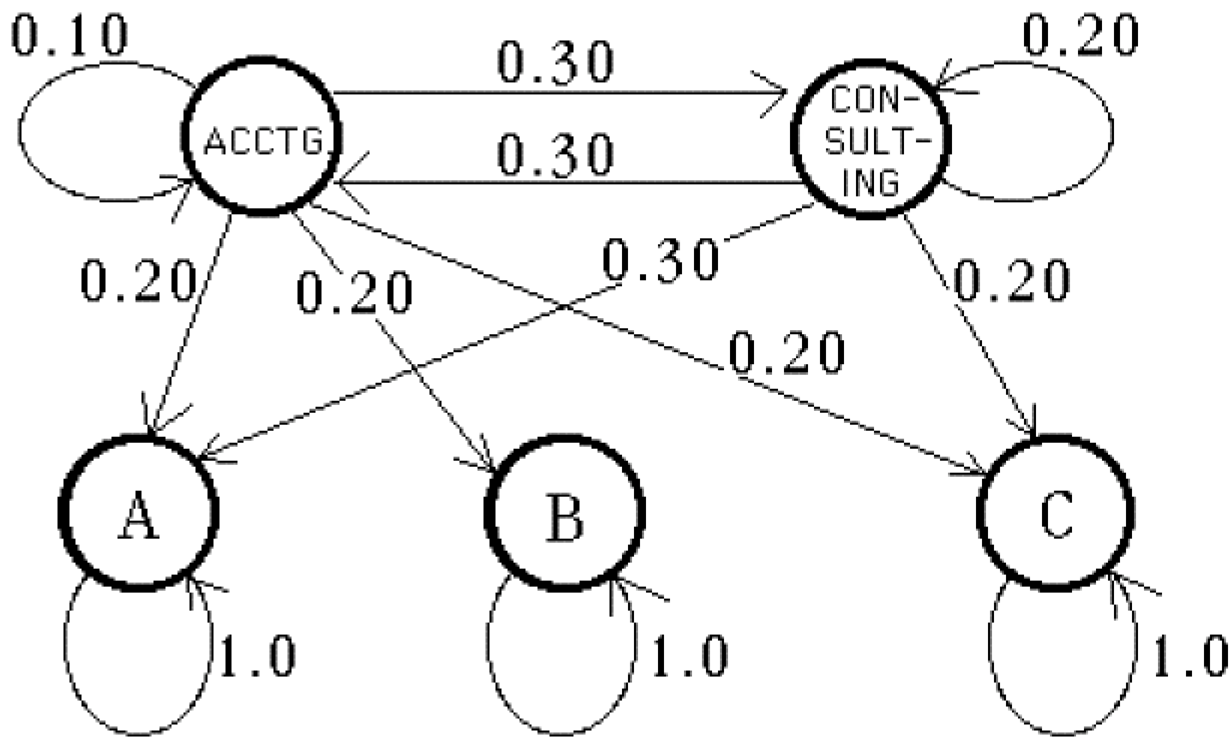
GM (Generous Motors) has three profit-making divisions (A, B, and C) which manufacture products, and two non-profit-making departments (Accounting and Management Consulting) which serve the profit-making divisions. During the past year, the budget for the Accounting Department was $6 million and for the Management Consulting Department it was $15 million.

For purposes of determining profitability, GM wants to charge the cost of the service departments to the manufacturing divisions. However, the efforts of the service departments were not uniformly distributed, and some of their efforts were directed toward serving their internal needs:

	Acctg	Mgmt Consulting	Division A	Division B	Division C
Accounting	10%	30%	20%	20%	20%
Mgmt Consulting	30%	20%	30%	0	20%

How should the costs be allocated?

Define a **Discrete-time Markov Chain** model of an invoice for $1 which is passed to the receiver of services by the provider, where the state is the current holder of the invoice:



States:

(1) Accounting Dept.
(2) Mgmt Consulting Dept.
(3) Manufacturing Division A
(4) Manufacturing Division B
(5) Manufacturing Division C

This "buck" originates when one of the two service departments receives $1 which has been budgeted for its operation, and is passed from one service department to the other until it is finally "absorbed" by one of the manufacturing divisions. *Note that the stages are not necessarily of equal time length!*

What is the probability that $1 originating in each of the two service departments is absorbed by Division A, etc.?

Transition Probability Matrix

	Acct Dept	Mgmt Consulting	Division A	Division B	Division C
Acct	0.10	0.30	0.20	0.20	0.20
Consulting	0.30	0.20	0.30	0	0.20
A	0	0	1	0	0
B	0	0	0	1	0
C	0	0	0	0	1

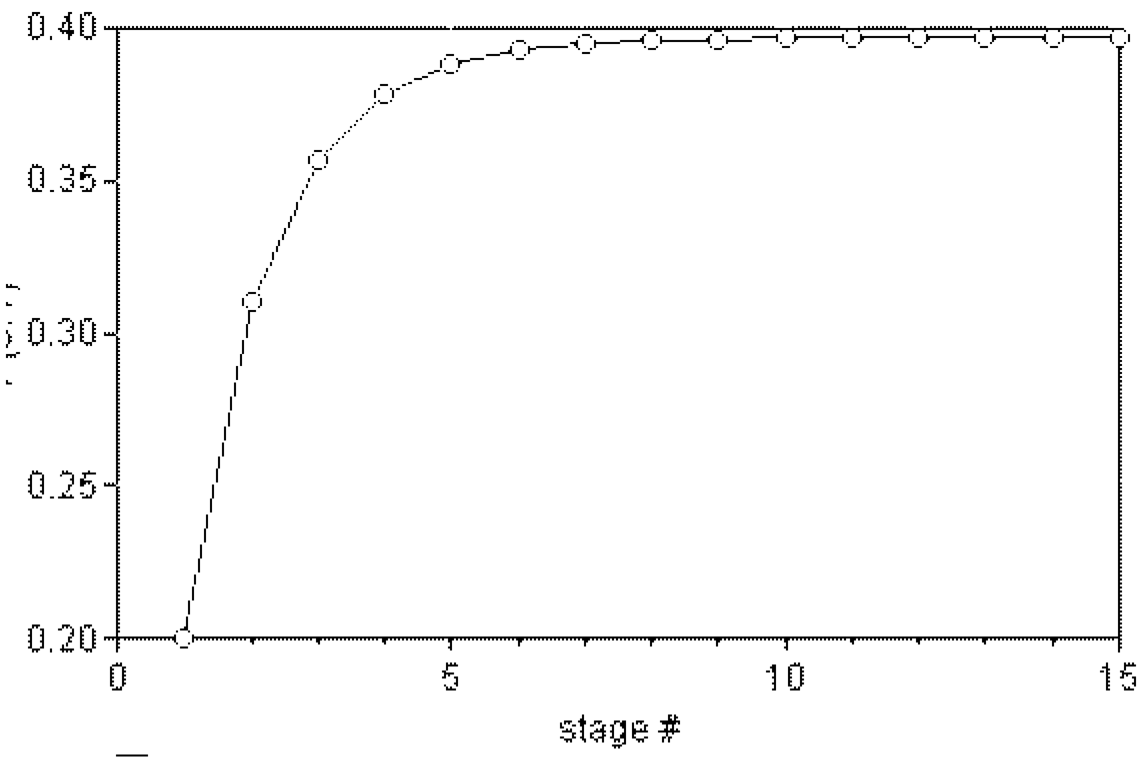
$$P = \begin{bmatrix} Q & R \\ 0 & I \end{bmatrix},$$

$$\text{where } Q = \begin{bmatrix} 0.1 & 0.3 \\ 0.3 & 0.2 \end{bmatrix} \text{ and } R = \begin{bmatrix} 0.2 & 0.2 & 0.2 \\ 0.3 & 0 & 0.2 \end{bmatrix}$$

$$E = (I - Q)^{-1} = \begin{bmatrix} 0.9 & -0.3 \\ -0.3 & 0.8 \end{bmatrix}^{-1} = \begin{bmatrix} 1.26984 & 0.47619 \\ 0.47619 & 1.42857 \end{bmatrix}$$

n-stage Transition Probabilities: For example, consider the n-stage transition probability $p_{1,3}^{(n)}$ that $1 originating in the Accounting Department is absorbed by Mfg. Division A:

n	$p_{1,3}^{(n)}$
1	0.2
2	0.31
3	0.357
4	0.3788
5	0.38863
6	0.393105
7	0.395136
8	0.396058
9	0.396477
10	0.396667
11	0.396754
12	0.396793
13	0.396811
14	0.396819



Absorption Probabilities

$$A = ER = \begin{bmatrix} 1.26984 & 0.47619 \\ 0.47619 & 1.42857 \end{bmatrix} \begin{bmatrix} 0.2 & 0.2 & 0.2 \\ 0.3 & 0 & 0.2 \end{bmatrix} = \begin{bmatrix} 0.3968 & 0.2540 & 0.3492 \\ 0.5238 & 0.0952 & 0.3810 \end{bmatrix}$$

For example, the probability that $1 originating in the Accounting Department is eventually absorbed by Manufacturing Division A is 39.68%, compared to 25.4% by Manufacturing Division B (even though each division uses the services of the Accounting Department equally!)

$$[6,15] \begin{bmatrix} 0.3968 & 0.2540 & 0.3492 \\ 0.5238 & 0.0952 & 0.3810 \end{bmatrix} = [10.2381,\ 2.95238, 7.80952]$$

That is, Division A should be allocated $10.2381 million of the cost, Division B $2.95238 million, and Division C $7.80952 million.