Transportation Simplex Method

Obtaining Initial Basic Feasible Solution



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Commonly-used methods:

- Northwest-Corner Method
- Least-Cost Method
- Vogel's Approximation Method (**VAM**)
- Russell's Method

Some of these give better quality initial solutions (which often requires fewer iterations of the simplex method in optimization) than others.

Common to all methods:

- Select a cell of the current tableau (i.e., a shipment route)
- Assign to that route the smaller of
- the available supply for the row
- the unsatisfied demand for the column
- Update the available supply & unsatisfied demand accordingly
- Cancel
  - ✓ row with no remaining supply *or*
  - ✓ column with no remaining demand
  - ✓ *but never both!*

Repeat these steps until no rows & columns remain.

The different methods differ only in their choice of a cell in the first step.

"Northwest-Corner" Method

- simplest to explain & use
- lower quality starting solution
- more appropriately called ***"Upper-left Corner"*** Method

Rule for selecting next cell to be assigned a shipment:

"Choose the upper-left corner of the remaining tableau"

- Because costs are ignored in the selection process, the resulting solution is generally of lower quality.

Example

(NW-Corner Method)

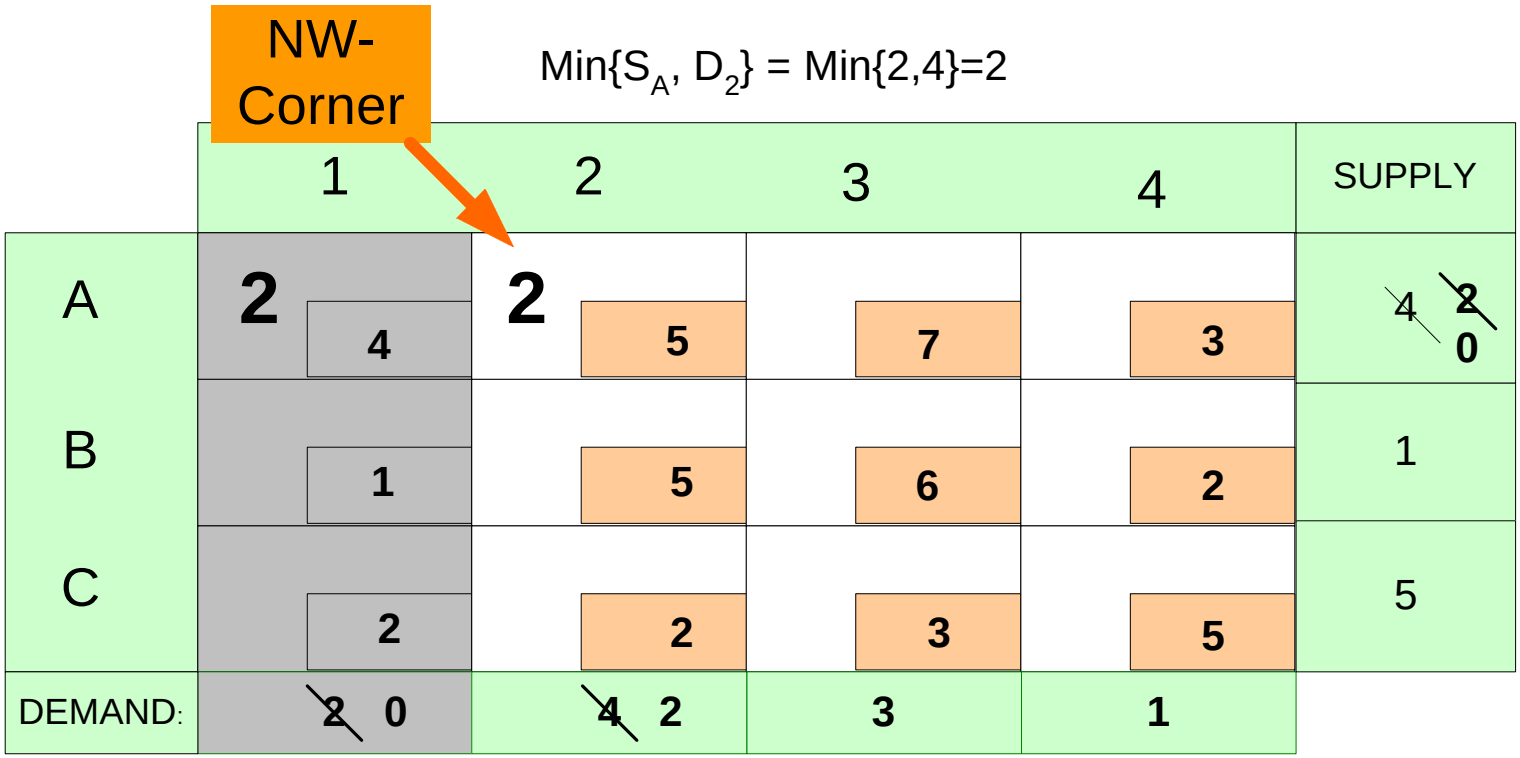
	1	2	3	4	SUPPLY
A	4	5	7	3	4
B	1	5	6	2	1
C	2	2	3	5	5
DEMAND:	2	4	3	1	

NW-Corner

$\text{Min}\{S_A, D_1\} = \text{Min}\{4,2\}=2$

	1	2	3	4	SUPPLY
A	**2** 4	5	7	3	~~4~~ 2
B	1	5	6	2	1
C	2	2	3	5	5
DEMAND:	~~2~~ 0	4	3	1	

The cell A1 is selected, and the largest shipment possible (2) is assigned. This satisfies the demand for destination #1, so the first column will now be ignored.



The "northwest corner" of the remaining tableau is cell A2.

We assign it a shipment which is the smaller of 2 (the remaining supply for A) and 4 (the demand for dstn#2).
We will now ignore row A, since the supply is now exhausted.

etc.

Least-Cost Method

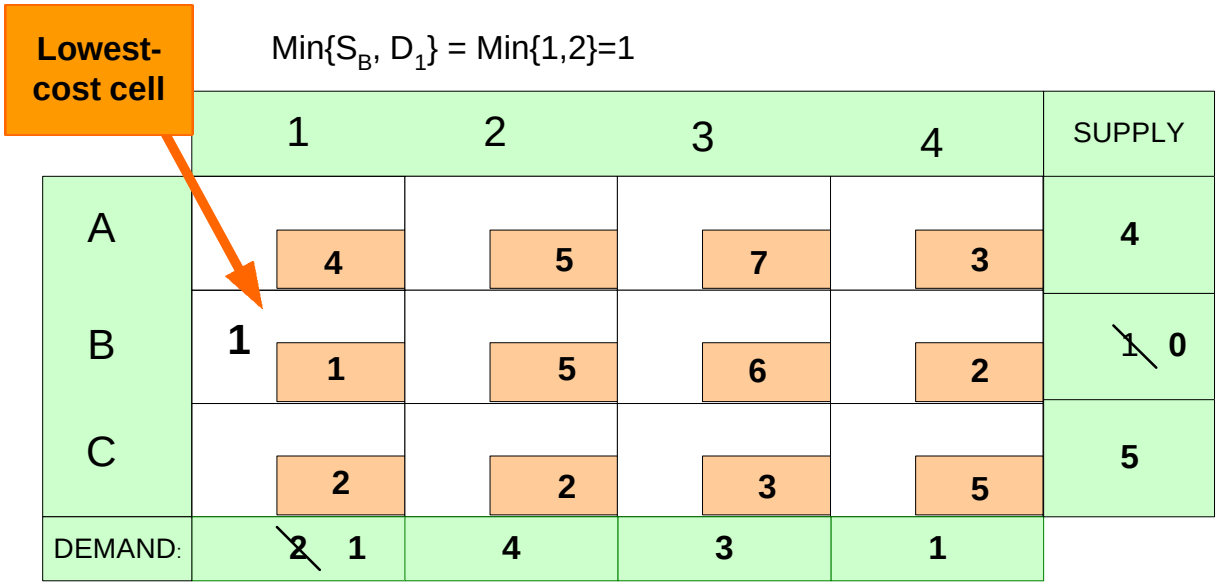
Rule for selecting next cell to be assigned a shipment:

> *"Choose the cell in the remaining tableau which has the lowest shipping cost"*
>
> *Note: ignore any row or column with all costs identical, e.g., a column for a dummy destination with zero costs.*

Example:

The lowest cost in the tableau is in cell B1, namely 1.

	1	2	3	4	SUPPLY
A	4	5	7	3	4
B	1	5	6	2	1
C	2	2	3	5	5
DEMAND:	2	4	3	1	



We assign the smaller of the available supply (1) and the unsatisfied demand (2).

Since the supply of B has been exhausted, that row will now be ignored.

Vogel's Approximation Method (VAM)

Rule for selecting next cell to be assigned a shipment:

1) associate with each row a penalty for *not* selecting the lowest cost in that row:
2) *namely, the difference between the lowest and 2nd-lowest costs*
3) associate with each column a penalty for *not* selecting the lowest column in that row.
4) **Identify the row or column with the largest penalty.**
5) **Select the lowest-cost cell in that row or column.**

The motivation is that, by means of this choice, the largest penalty will be avoided!

Example:

The penalty for a row or column is the difference between the two smallest costs.

	1	2	3	4	SUPPLY
A	4	5	7	3	4
B	1	5	6	2	1
C	2	2	3	5	5
DEMAND:	2	4	3	1	

Row penalties		Column penalties:	2 – 1 = 1	5 – 2 = 3	6 – 3 = 3	3 – 2 = 1	
			1	2	3	4	SUPPLY
4 – 3 = 1	A		4	5	7	3	4
2 – 1 = 1	B		1	5	6	2	1
3 – 2 = 1	C		2	2	3	5	~~5~~ 2
	DEMAND:		2	4	~~3~~ 0	1	

3

Minimum cost in column

$\text{Min}\{S_C, D_3\} = \text{Min}\{5,3\}=3$

The largest penalty is in the column for destination #3. The smallest cost in that column is in cell **C3**. By choosing this cell, we avoid having to use cell **B3** with a cost of 6 *(an increase of 3)*!

Russell's Method

Motivation: approximate the ***dual variables***

$\bar{U}_i$ for each source ***i*** &

$\bar{V}_j$ for each destination ***j***

in order to approximate the reduced cost

$\Delta_{ij} = C_{ij} - \left(\bar{U}_i + \bar{V}_j\right)$ of each cell (i,j).

The cell with the "most negative" Δ_{ij} is then selected.

The "approximations":

$\bar{U}_i$ = largest cost in row i (ignoring those columns whose demands are already satisfied)

$\bar{V}_j$ = largest cost in column j (ignoring those rows whose supplies are already exhausted)

Example:

We first compute the estimates of the dual variables, and then the estimated "reduced cost".

	1	2	3	4	SUPPLY
A	4	5	7	3	4
B	1	5	6	2	1
C	2	2	3	5	5
DEMAND:	2	4	3	1	

$\bar{U}_i$		$\bar{V}_j$ = 4	5	7	5	
		1	2	3	4	SUPPLY
7	A	Δ=4–(7+4)= –7 **4**	Δ=5–(7+5)= –7 **5**	Δ=7–(7+7)= –7 **7**	Δ=3–(7+4)= –8 **3**	4
6	B	**Δ=1–(6+4)= –9** **1**	Δ=5–(6+5)= –6 **5**	Δ=6–(6+7)= –6 **6**	Δ=2–(6+4)= –8 **2**	1
5	C	Δ=2–(5+4)= –7 **2**	Δ=2–(5+5)= –8 **2**	**Δ=3–(5+7)= –9** **3**	Δ=5–(5+4)= –4 **5**	~~5~~ 2
	DEMAND:	2	4	~~3~~ 0	1	

Minimum cost in column

$\text{Min}\{S_C, D_3\} = \text{Min}\{5,3\}=3$

The smallest (i.e., **"most negative"**) Δ_{ij} is **–9**, in both cells **B1** and **C3**. (In this example, we have arbitrarily chosen cell C3.)