Design of Coal Slurry Pipeline

Geometric Programming Model

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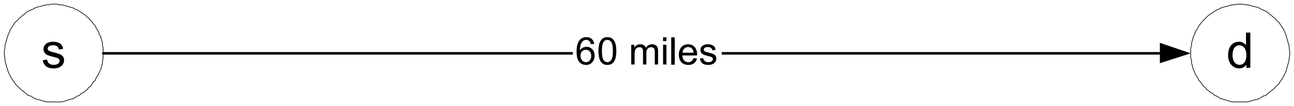
Certain solid materials (e.g., coal, wood pulp, mineral ore) can be transported in pipelines while suspended in fluids (e.g., water or crude oil). The suspension is known as a ***slurry***.

Such a system is to be designed to transport coal from a mine *s* to a destination *d* which is 60 miles distant and 300 feet lower in altitude.



The coal must be pulverized at the mine, suspended in water (which is freely available). The slurry is then to be pumped through the pipeline at a rate of 10 tons/hour.

At the final destination, the coal must be retrieved from the slurry and the water treated before it is discharged into a body of water.

DATA

Length of pipeline = **L** = 60 miles

Change in elevation = **h** = –300 feet

Required capacity = **K** = 10 tons of coal per hour

Specific gravity of coal = **G** = 1.3 *(i.e. density of coal = 1.3×density of water)*

Installed cost of pipe ($/ft) including right-of-way:

$$C_1\left(D\right) = 1 + 0.03D^{5/2}$$

where **D** is the inside diameter (inches)

Installed cost of pumping stations:

$$C_p = \$250/hp \text{ of pump capacity}$$

Installed cost of system for injecting coal at beginning of line and separating at end of line

$$C_{is} = \$50/ton \quad \text{capacity/day}$$

Cost of water treatment at outfall

$$C_t = \$100/million \text{ gallons}$$

Electrical energy cost:

$$C_e = \$0.01/kilowatt - hr$$

Efficiency of electric motor/pump combination: 60%

Required Horsepower delivered by pumps

$$HP = \frac{W_w + W_c}{550}\left(h + H\right)$$

where

W_w = weight of water delivered per second
W_c = weight of coal delivered per second
H = total head loss (feet) due to friction

Note:

- *a "head" of 1 foot = pressure at bottom of a column of water 1 foot in depth.*
- *1 h.p. = 550 ft-lb/sec.*

Friction in the pipe causes a loss of pressure, i.e., pressure must be applied at the inlet of the pipeline to overcome the friction and cause the slurry to flow.

$$H = 80\frac{V^2}{D}\left(1+1.4c\right) \quad per\ mile$$

where

V = flow velocity (ft/second)

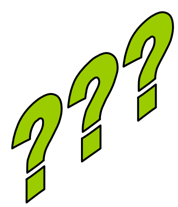
c = coal concentration (% of total weight of slurry)

Assume

- operation is continuous, 24 hr/day, 7 days/wk.
- all components have a 30-year lifetime with zero salvage value.
- rate of return for computing time value of money: 10% per annum.

What is the optimal design of the system, specifically,

- c = concentration of coal
- D = diameter of pipe



Design Variables:

symbol	definition	units
D	diameter	inches
c	% coal by weight	
V	flow velocity	ft/sec
W_w	flow rate of water	lb/sec
W_c	flow rate of coal	lb/sec
H	head loss due to friction	ft
P	power delivered by pump	hp

Constraints:

$$\frac{W_c}{W_c + W_w} = c$$ *Concentration (by weight) of slurry*

$$W_c = 10\frac{tons}{hr} \times \frac{2000\ lb}{ton} \times \frac{1\ hr}{3600\ \sec} = \frac{50}{9}\frac{lb}{\sec}$$

Required capacity of pipeline

Flow rate = cross-sectional area × flow velocity

$$\frac{W_w\left({lb}/{\sec}\right)}{62.5\left({lb}/{ft^3}\right)}+\frac{W_c\left({lb}/{\sec}\right)}{1.3\times 62.5\left({lb}/{ft^3}\right)}=\frac{\pi D^2}{4}\left(in^2\right)\times\frac{1\,ft^2}{144 in^2}\times V\left(\frac{ft}{\sec}\right)$$

$$\Rightarrow\left(0.016W_w+0.0123W_c\right)\frac{ft^3}{\sec}=5.454\times 10^{-3}D^2V\,\frac{ft^3}{\sec}$$

$$H = \frac{80V^2}{D}\left(1+1.4c\right)$$ *Required head (pressure) to overcome friction*

$$P \geq \frac{W_w + W_c}{550}\left(H - 300\right)$$ *Horsepower required to provide head (pressure)*

Costs:

Capital costs

Pipe: $\left(1+0.03D^{5/2}\right)\frac{\$}{ft}\times 60mi.\times\frac{5280\,ft}{mi}$

Pumping station: $250\frac{\$}{hp}\times P\ hp$

(capital recovery factor (i=10%, n=30 yr) is 0.10608)

Operating costs

Energy: $0.05\frac{\$}{kw\times hr}\times 1.34\frac{kw}{hp}\times\frac{24hr}{day}\times\frac{365days}{yr}\times\frac{P}{0.6}hp = 9.782P\frac{\$}{yr}$

Water treatment:

$$\frac{100}{10^6}\frac{\$}{gal.}\times\frac{7.48gal.}{ft^3}\times\frac{W_w}{62.5}\frac{lb/\sec}{lb/ft^3}\times 3.1536\times 10^7\frac{\sec}{yr} = 377.42W_w\frac{\$}{yr}$$

$$\textit{Minimize} \quad 1008.18D^{5/2} + 36.302P + 377.42W_w$$

s.t.

$$\frac{W_c}{W_w + W_c} = c$$

$$W_c = \frac{50}{9}$$

$$0.016W_w + 0.0123W_c = 5.454 \times 10^{-3} D^2 V$$

$$H = \frac{80V^2}{D}(1 + 1.4c)$$

$$P \geq \frac{W_w + W_c}{550}(H - 300)$$

$$D, C, V, W_w, W_c, H, P > 0$$

(Note that W_c may be eliminated by substituting $W_c = {}^{50}/_{9}$.)

Can we formulate this problem in standard GP form?

Consider the equation: $\frac{W_c}{W_w + W_c} = c$

Which type of inquality would be tight at the optimum?

$$\frac{W_c}{W_w + W_c} \le c \Rightarrow W_c \le cW_w + cW_c \Rightarrow W_c - cW_w \le cW_c$$

$$\Rightarrow \qquad c^{-1} - W_w W_c^{-1} \le 1 \quad \textit{Signomial constraint}$$

$$0.016W_w + 0.0123W_c \le 5.454 \times 10^{-3} D^2 V$$

$$\Rightarrow \qquad 2.9336W_w D^{-2} V^{-1} + 2.2552 W_c D^{-2} V^{-1} \le 1$$

$$H \ge \frac{80V^2}{D}\left(1 + 1.4c\right)$$

$$\Rightarrow \qquad 80V^2 D^{-1} H^{-1} + 112V^2 D^{-1} H^{-1} c \le 1 \quad \textit{Posynomial} \quad \textit{constraint}$$

$$P \geq \frac{W_w + W_c}{550}\left(H - 300\right)$$

$$\Rightarrow \qquad 0.001818 W_w P^{-1} h - 0.001818 W_C P^{-1} H - 0.5454 W_w P^{-1} - 0.5454 W_c P^{-1} \leq 1$$

Signomial constraint

$$\textit{Minimize}\ \ 1008.18D^{5/2} + 36.302P + 377.42W_w$$

s.t.

$$c^{-1} - W_w W_c^{-1} \le 1$$

$$\frac{50}{9} W_c^{-1} \le 1$$

$$2.9336 W_w D^{-2} V^{-1} + 2.2552 W_c D^{-2} V^{-1} \le 1$$

$$80 V^2 D^{-1} H^{-1} + 112 V^2 D^{-1} H^{-1} c \le 1$$

$$0.001818 W_w P^{-1} h - 0.001818 W_C P^{-1} H - 0.5454 W_w P^{-1} - 0.5454 W_c P^{-1} \le 1$$

$$D, C, V, W_w, W_c, H, P > 0$$

Signomial GP

- *T=# terms = 14*
- *N=#variables = 7*
- *Degrees of difficulty: 6*

Suppose we arbitrarily assign a value to the coal concentration, **c**.

For example, let c = 50%, so $W_w = W_c = \frac{50}{9}$

Then

$$\left(0.016W_w + 0.0123W_c\right)\frac{ft^3}{\sec} = 5.454\times 10^{-3} D^2 V \frac{ft^3}{\sec}$$

$$0.15722\frac{ft^3}{\sec} \leq 5.454\times 10^{-3} D^2 V \frac{ft^3}{\sec}$$

$$\Rightarrow \qquad 28.827 D^{-2} V^{-1} \leq 1$$

Also,

$$H = \frac{80V^2}{D}\left(1+1.4c\right) = 136\frac{V^2}{D}$$

and so $P \geq \frac{W_w + W_c}{550}\left(H - 300\right) = \frac{\frac{50}{9}+\frac{50}{9}}{550}\left(136\frac{V^2}{D} - 300\right) = 2.7474\frac{V^2}{D} - 6.0606$

The problem reduces to

Minimize $1008.18D^{5/2} + 99.736V^2D^{-1}$ — $36.302 \times 6.06 + 377.42$ *constant!*

s.t.

$$28.827D^{-2}V^{-1} \le 1$$

which is a **posynomial GP** problem with **ZERO** degree of difficulty!

Dual solution is $\delta_{01}^* = 2/3, \delta_{02}^* = 1/3, \delta_{11}^* = \lambda^* = 2/3$

This is determined by the exponents alone, which don't depend upon the value of the concentration c!

This means that, for any concentration c, the pipe cost should be twice the cost (initial + operating) of the pump!

For c=50%, the minimum cost of pipe & pump is

$$\left(\frac{1008.18}{2/3}\right)^{2/3}\left(\frac{99.736}{1/3}\right)^{1/3}\left(2/3\right)^{2/3}=(1512.27)^{2/3}(299.21)^{1/3}(0.66667)^{2/3}$$

$$=131.75\times 6.6884\times 0.76314=672.49\left(\$/yr\right)$$

Add to this the **water treatment cost** $377.42W_w=377.42\times\frac{50}{9}=2096.78\left(\$/yr\right)$

and subtract the **constant term** $36.302\times 6.0606=220.01\left(\$/yr\right)$

Total: $2549.26 /year

This suggests that we can write the optimum of the zero degree-of-difficulty problem as a function of the concentration c! We would then be able to solve the problem by a ***one-dimensional*** search algorithm.

For fixed values of c, the GP dual solution is

$$\delta_{01}^{*} = 2/3, \delta_{02}^{*} = 1/3, \delta_{11}^{*} = \lambda^{*} = 2/3$$

and the optimal value is

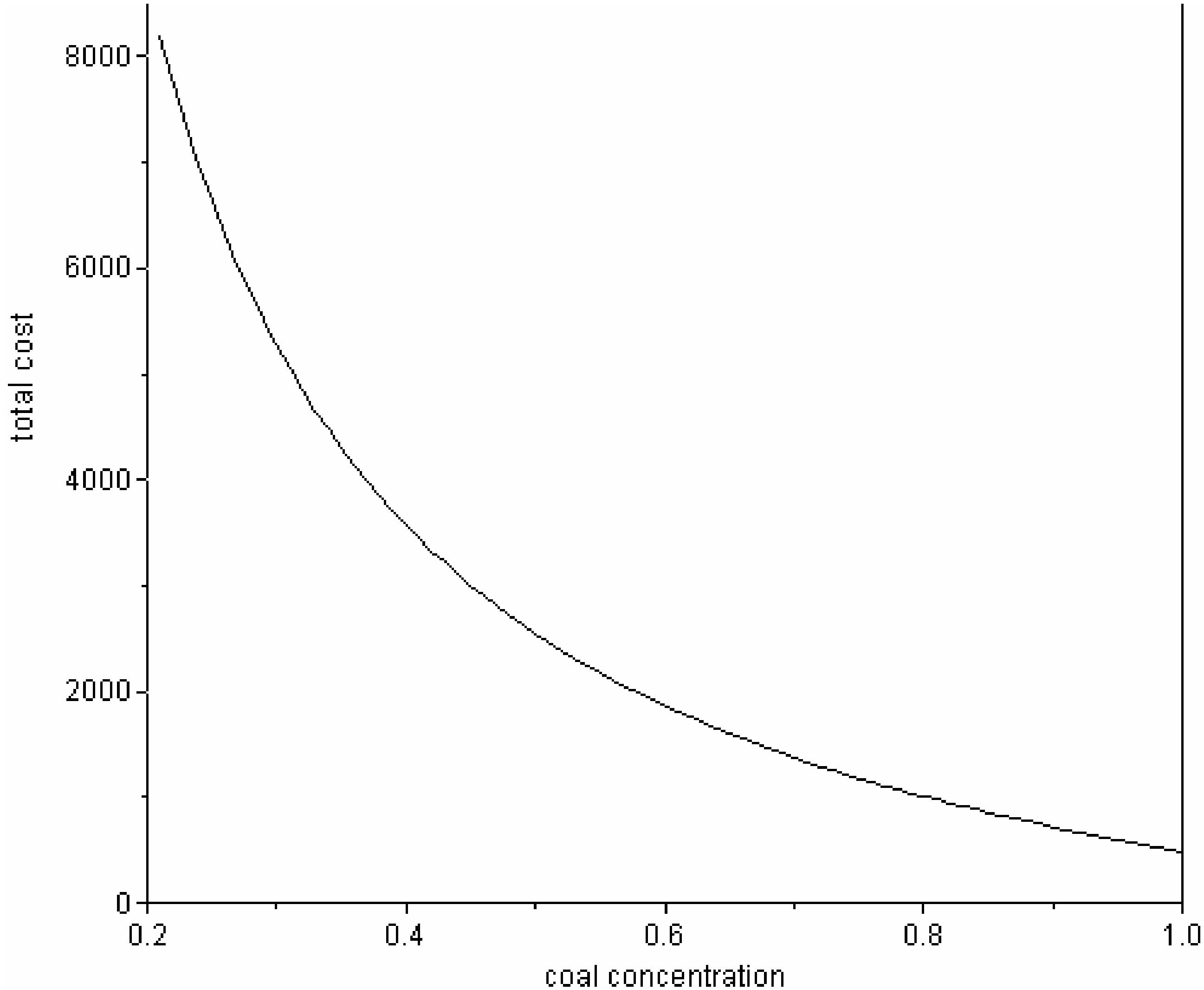
$$\Phi(c) = \left(\frac{1008.18}{2/3}\right)^{2/3} \left(\frac{\frac{29.335}{c} + 41.069}{1/3}\right)^{1/3} \left(2/3\right)^{2/3} + \frac{1986}{C} - 2096$$

$$\Phi(c) = 131.75\left(\frac{88}{c} + 123.2\right)^{1/3} (0.76314) + \frac{1986}{c}$$

$$\Phi(c) = 100.54\left(\frac{88}{c} + 123.2\right)^{1/3} + \frac{1986}{c} - 2096$$

= minimal attainable cost when concentraction c is used.

Examination of the cost function shows that it is monotonically decreasing:



It is obvious, however, that c=100% is not feasible in practice, i.e., we have omitted a constraint which limits the concentration of coal so that the slurry will behave as a fluid!

Optimal Design as a function of the concentration c:

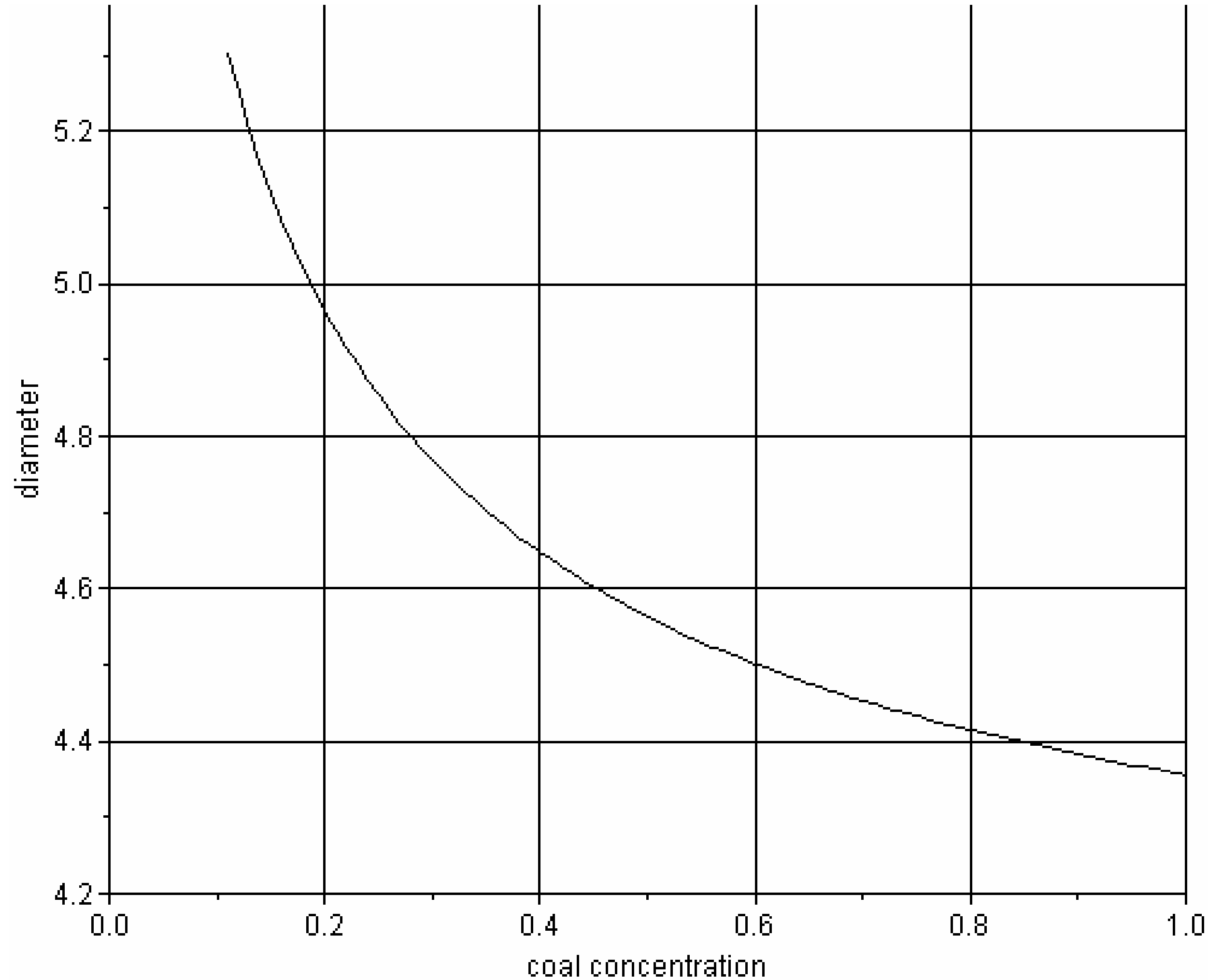
$$1008.18 D^{5/2} = \delta_1^* \left[100.54 \left(\frac{88}{c} + 123.2 \right)^{1/3} \right]$$

$$\Rightarrow \qquad D^* = \left[6.64 \left(\frac{88}{c} + 123.2 \right)^{1/3} \right]^{2/5}$$

$$\left(\frac{29.335}{c} + 41.069 \right) V^2 D^{-1} = \delta_2^* \left[100.54 \left(\frac{88}{c} + 123.2 \right)^{1/3} \right]$$

$$\Rightarrow \qquad V^* = \left[\frac{33.513 \left(\frac{88}{c} + 123.2 \right)^{1/3} D^*}{\frac{29.335}{c} + 41.069} \right]^{1/2}$$

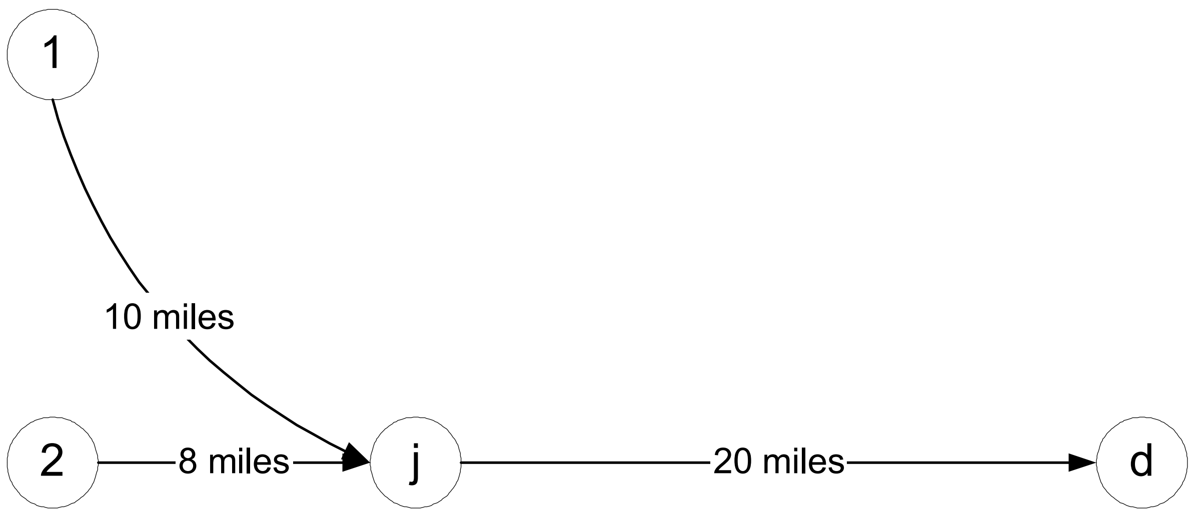
Optimal diameter as a function of coal concentration:



Assume that coal is to be pumped from two sources to a common discharge point. The required capacities are 9 tons of coal per hour from source #1 and 6 tons/hour from source #2. Pumps are to be installed only at the two sources, and not at the junction of the two pipelines. The slurry concentration will be 50% coal (by weight).

Elevations:

- source #1: 1500 ft
- source #2: 1200 ft
- junction j: 1000 ft
- discharge d: 600 ft



Find the optimal pipe diameters in the three links *(assuming diameters are constant within each link)*.