Markov Chain Analysis of Insurance Premiums

AllCity Insurance Company sets its automobile insurance premiums based upon the insured's accident history:

Premium	History
$100	No accident during past 2 years
$300	Accident during only one of past 2 years
$400	Accident during both of past 2 years

Analysis of historical data implies that
- a driver who has had an accident during the current year has a **10%** probability of having another accident next year.

- a driver who has not had an accident during the current year has a **3%** probability of having an accident next year.

What is the average premium paid by a driver?

If we were to define a Markov chain model with three states (one per premium cost), the Markov property would be violated, i.e., the process would not be memoryless! Therefore, the states must be defined so that it includes all information upon which future behavior is dependent.

Stages are defined such that at the anniversary of the policy, a driver is classified into one of **four** states:

(1) No accidents during the current & previous year
(2) Accident during both current & previous years
(3) Accident during current year, but not previous year
(4) Accident during previous year, but not current year
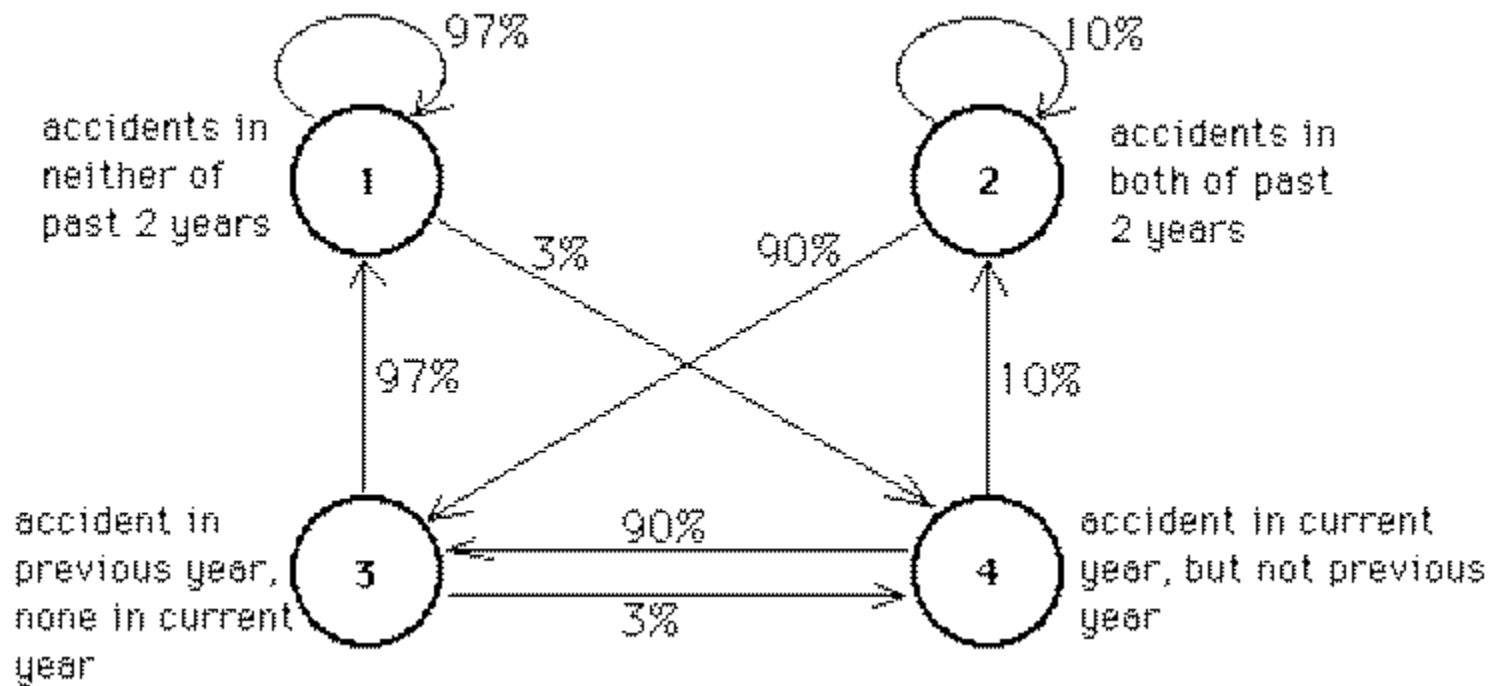
accidents in
neither of   (1)
past 2 years

(2)   accidents in
        both of past
        2 years

accident in
previous year,   (3)
none in current
year

(4)   accident in current
        year, but not previous
        year

97%

accidents in
neither of
past 2 years

1

3%

90%

10%

accidents in
both of past
2 years

2

97%

10%

accident in
previous year,
none in current
year

3

90%

3%

accident in current
year, but not previous
year

4

Steadystate Distribution

State	Premium	Probability	
1	$100	0.93870968	93.870968
2	$400	0.00322581	1.2903226
3	$300	0.0290323	8.7096774
4	$300	0.0290323	8.7096774
		SUM:	**$112.58**

The average premium is $112.58.

Mean First Passage Times

	ONE	TWO	THREE	FOUR
ONE	1.0652921	343.33333	34.444444	33.333333
TWO	2.1764032	310	1.1111111	34.444444
THREE	1.0652921	343.33333	34.444444	33.333333
FOUR	2.1764032	310	1.1111111	34.444444