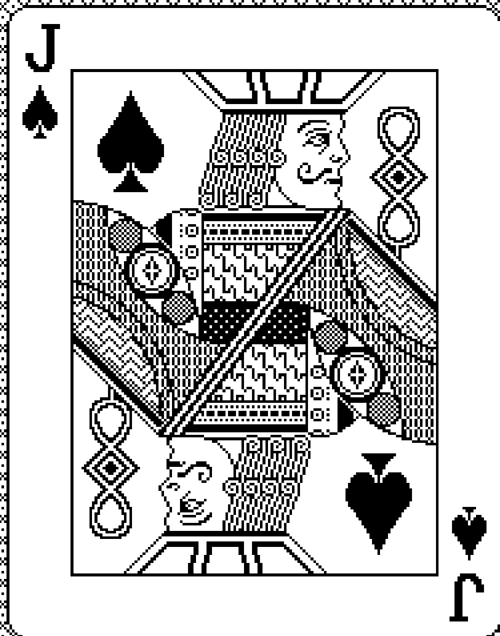

The Casino Problem

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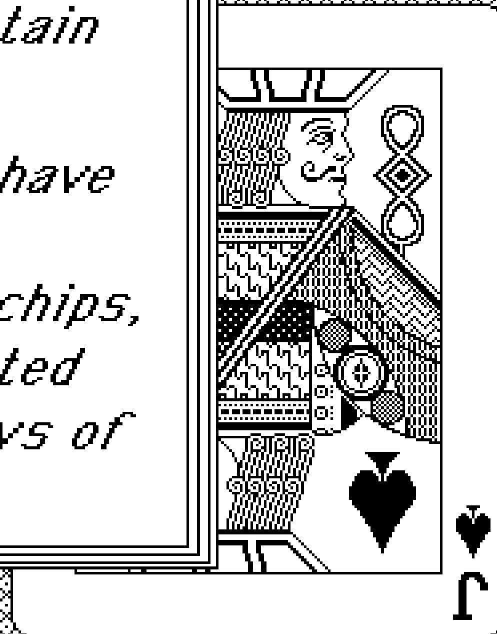
A dynamic young programmer believes that he has developed a system for winning a certain game at the casino.

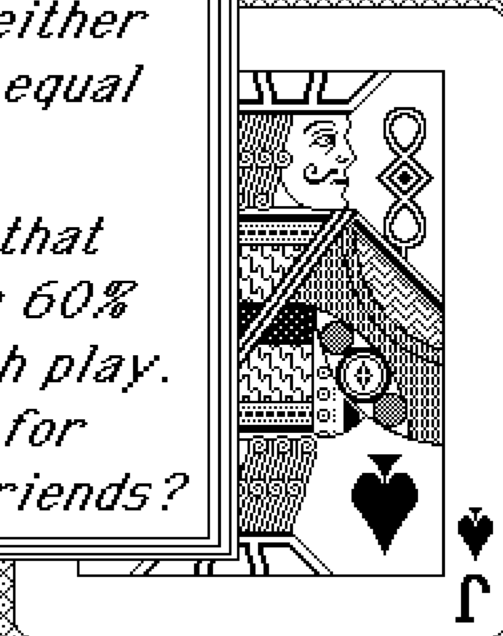
His friends doubt this, and have made a large bet with him:

> *that, starting with three chips, he will not have accumulated five chips after three plays of the game.*



Each play of the game involves betting any desired number of one's available chips, and either losing them or winning an equal number.

The programmer believes that his system will give him a 60% probability of winning each play. What is the best strategy for winning the bet with his friends?



Stage: n = play of the game

State: S_n = # of chips accumulated by the beginning of play #n $(0 \le S_n \le 5)$

Decision: X_n = # of chips to be bet on play #n of the game $(0 \le X_n \le S_n)$

Optimal value function:

$V_n(S_n)$ = maximum probability that he accumulates at least 5 chips, given that before play #n he has S_n chips.

Recursive Definition

p = probability of winning a play of game = 60%

For n=1, 2, & 3:

$$V_n(S_n) = \underset{0 \le X_n \le S_n}{\text{Maximum}} \{ p V_{n+1}(S_n + X_n) + (1-p) V_{n+1}(S_n - X_n) \}$$

- p: p{win}
- $S_n + X_n$: total chips if he wins
- $(1-p)$: p{loss}
- $S_n - X_n$: total chips if he loses

$$V_4(S_4) = \begin{cases} 1 & \text{if } S_4 \ge T \\ 0 & \text{if } S_4 < T \end{cases}$$

T = target = 5

```
∇VALUE←F N;t
 ⍝
 ⍝          Optimal Value Function of DP model
 ⍝             of the Casino Problem
 ⍝
 →LAST IF N=4

 ⍝             Evaluate Optimal Value Function

 VALUE←P MAX∆E  (F N+1)[TRANSITION s ∘.+ x ∘.× d]
 →0

 ⍝   After last play, return 1 if target is achieved,
 ⍝            else return 0

 LAST:VALUE←(s ≥ TARGET),-BIG
∇
```

Stage 3

s \ x	0	1	2	3	4	5
0	0.00	¯99.99	¯99.99	¯99.99	¯99.99	¯99.99
1	0.00	0.00	¯99.99	¯99.99	¯99.99	¯99.99
2	0.00	0.00	0.00	¯99.99	¯99.99	¯99.99
3	0.00	0.00	0.60	¯99.99	¯99.99	¯99.99
4	0.00	0.60	¯99.99	¯99.99	¯99.99	¯99.99
5	1.00	¯99.99	¯99.99	¯99.99	¯99.99	¯99.99

Stage 3

s \ x	0	1	2	3	4	5
0	0.00	¯99.99	¯99.99	¯99.99	¯99.99	¯99.99
1	0.00	0.00	¯99.99	¯99.99	¯99.99	¯99.99
2	0.00	0.00	0.00	¯99.99	¯99.99	¯99.99
3	0.00	0.00	0.60	¯99.99	¯99.99	¯99.99
4	0.00	0.60	¯99.99	¯99.99	¯99.99	¯99.99
5	1.00	¯99.99	¯99.99	¯99.99	¯99.99	¯99.99

State	Optimal Values	Optimal Decisions
0	0.00	0
1	0.00	0
		1
2	0.00	0
		1
		2
3	0.60	2
4	0.60	1
5	1.00	0

Stage 2

s \ x	0	1	2	3	4	5
0	0.00	⁻99.99	⁻99.99	⁻99.99	⁻99.99	⁻99.99
1	0.00	0.00	⁻99.99	⁻99.99	⁻99.99	⁻99.99
2	0.00	0.36	0.36	⁻99.99	⁻99.99	⁻99.99
3	0.60	0.36	0.60	⁻99.99	⁻99.99	⁻99.99
4	0.60	0.84	⁻99.99	⁻99.99	⁻99.99	⁻99.99
5	1.00	⁻99.99	⁻99.99	⁻99.99	⁻99.99	⁻99.99

Stage 2

s \ x	0	1	2	3	4	5
0	0.00	¯99.99	¯99.99	¯99.99	¯99.99	¯99.99
1	0.00	0.00	¯99.99	¯99.99	¯99.99	¯99.99
2	0.00	0.36	0.36	¯99.99	¯99.99	¯99.99
3	0.60	0.36	0.60	¯99.99	¯99.99	¯99.99
4	0.60	0.84	¯99.99	¯99.99	¯99.99	¯99.99
5	1.00	¯99.99	¯99.99	¯99.99	¯99.99	¯99.99

State	Optimal Values	Optimal Decisions
0	0.00	0
1	0.00	0
		1
2	0.36	1
		2
3	0.60	0
		2
4	0.84	1
5	1.00	0

Stage 1

s \ x	0	1	2	3	4	5
0	0.00	¯99.99	¯99.99	¯99.99	¯99.99	¯99.99
1	0.00	0.22	¯99.99	¯99.99	¯99.99	¯99.99
2	0.36	0.36	.50	¯99.99	¯99.99	¯99.99
3	0.60	0.65	.60	¯99.99	¯99.99	¯99.99
4	0.84	0.84	¯99.99	¯99.99	¯99.99	¯99.99
5	1.00	¯99.99	¯99.99	¯99.99	¯99.99	¯99.99

Stage 1

s \ x	0	1	2	3	4	5
0	0.00	¯99.99	¯99.99	¯99.99	¯99.99	¯99.99
1	0.00	0.22	¯99.99	¯99.99	¯99.99	¯99.99
2	0.36	0.36	.50	¯99.99	¯99.99	¯99.99
3	0.60	0.65	.60	¯99.99	¯99.99	¯99.99
4	0.84	0.84	¯99.99	¯99.99	¯99.99	¯99.99
5	1.00	¯99.99	¯99.99	¯99.99	¯99.99	¯99.99

We see that (assuming that p=60%) he has a 65% probability of winning the bet with his friends.

State	Optimal Values	Optimal Decisions
0	0.00	0
1	0.22	1
2	0.50	2
3	0.65	1
4	0.84	0 1
5	1.00	0

Optimal Returns and Decisions

Stage 1

State	Optimal Values	Optimal Decisions
0	0.00	0
1	0.22	1
2	0.50	2
3	0.65	1
4	0.84	0
		1
5	1.00	0

Stage 2

State	Optimal Values	Optimal Decisions
0	0.00	0
1	0.00	0
		1
2	0.36	1
		2
3	0.60	0
		2
4	0.84	1
5	1.00	0

Stage 3

State	Optimal Values	Optimal Decisions
0	0.00	0
1	0.00	0
		1
2	0.00	0
		1
		2
3	0.60	2
4	0.60	1
5	1.00	0