[Posynomial] Geometric Programming

The general primal problem of geometric programming (GP) [Duffin et al.] is to

$$\begin{aligned}&\text{Minimize } g_0(x)\\&\text{subject to } g_i(x) \le 1,\ i=1, 2, ..., m\\&x > 0\end{aligned} \tag{1}$$

where the functions g_i are *posynomials*, i.e.,

$$g_i(x) = \sum_{j=1}^{T_i} c_{ij} \prod_{n=1}^{N} x_n^{a_{ijn}} \tag{2}$$

The exponents a_{ijn} are arbitrary real numbers, but the coefficients c_{ij} are assumed to be positive constants and the decision variables x_n are required to be strictly positive. The corresponding posynomial GP dual problem is to

$$\text{Maximize } v(\delta, \lambda) = \prod_{i=0}^{m} \prod_{j=1}^{T_i} \left(\frac{c_{ij}\lambda_i}{\delta_{ij}}\right)^{\delta_{ij}} \tag{3}$$

$$\text{subject to } \sum_{i=0}^{m} \sum_{j=1}^{T_i} a_{ijn}\delta_{ij} = 0,\ n=1, 2, ..., N \tag{4}$$

$$\lambda_i = \sum_{j=1}^{T_i} \delta_{ij},\ i=0, 2, ..., m \tag{5}$$

$$\lambda_0 = 1 \tag{6}$$

$$\delta_{ij} \ge 0,\ j=1,2,...,T_i,\ i=0,1,...,m \tag{7}$$

This dual problem offers several computational advantages: after using (5) to eliminate λ, the logarithm of the objective (3) is a concave function to be maximized over a linear system. This linear system has T variables, where $T=T_0 + T_1 + ... + T_m$, and N+1 equations, and hence T-(N+1) is referred to as its *degree of difficulty*. If an optimal dual solution (δ^*,λ^*) is known, then the following relationships may be used to compute a primal solution x^* in nonpathological cases:

$$\delta_{ij}^* \, g_i\left(x^*\right) = \lambda_i^* \, c_{ij} \prod_{n=1}^{N} x^{*\,a_{ijn}}_n,\ \ j=1, 2, ..., T_i,\ i=0, 1, ..., m \tag{8}$$

where $g_0(x^*)=v(\delta^*,\lambda^*)$ and, for $i>0$, $g_i(x^*)=1$ if $\lambda_i \ge 0$. Note that, from these relationships, one may obtain a system of equations linear in the logarithms of the optimal values of the primal variables:

$$\sum_{n=1}^{N} a_{ijn} \ln x_n = \ln\left(\frac{\delta_{ij}^* g_i(x^*)}{\lambda_i^* c_{ij}}\right), \quad j=1, 2, ..., T_i \text{ for each } i=0, 1, ..., m \text{ such that } \lambda_i^* \neq 0 \quad (9)$$

Typically, but not always, the system of linear equations (9) uniquely determines the optimal x*. (Cf. [Dembo] for a discussion of the recovery of primal solutions from the dual solution in general.)

References

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