ststst 56:171 Operations Research tststs

tststs Midterm Examination Solutions ststst

ststst Fall 1997 tststs

Answer both questions of Part One, and 4 (out of 5) problems from Part Two.

		Possible
Part One:	1. True/False	15
	2. Sensitivity analysis (LINDO)	25
Part Two:	3. Simplex method	15
	4. LP duality	15
	5. Transportation problem	15
	6. Project scheduling	15
	7. Decision analysis	15
	total possible:	100

Max achieved:	97
Mean:	80.25
Std. Deviation:	11.37
Median:	82

tststs PART ONE tststs

(1.) ***True/False:*** Indicate by "+" or "o" whether each statement is "true" or "false", respectively:

__+_ a. If there is a tie in the "minimum-ratio test" of the simplex method, the next basic solution will be degenerate.

__o_ b. "Crashing" a critical path problem is a technique used to find a good initial feasible solution. *Note: Crashing a project means allocating resources to reduce its duration.*

__+_ c. In the two-phase simplex method, an artificial variable is defined for each constraint row lacking a slack variable (assuming the right-hand-side of the LP tableau is nonnegative).

__+_ d. If the primal LP feasible region is nonempty and unbounded, then the dual LP is infeasible.

__o_ e. In PERT, the total completion time of the project is assumed to have a BETA distribution. *Note: The completion time is assumed to have a Normal dist'n, based on central limit theorem.*

__o_ f. The Revised Simplex Method, for most LP problems, requires fewer pivots than the ordinary simplex method. *Note: the number of pivots should be the same; the computations within each iteration are performed differently.*

__+_ g. All tasks on the critical path of a project schedule have their latest start time equal to their earliest start time.

__+_ h. When maximizing in the simplex method, the value of the objective function increases at every iteration unless a degenerate tableau is encountered.

__o_ i. The critical path in a project network is the shortest path from a specified source node (beginning of project) to a specified destination node (end of project). *Note: the critical path is the longest path!*

__+_ j. The assignment problem is a special case of a transportation problem.

__o_ k. If you make a mistake in choosing the pivot column in the simplex method, the next basic solution will be infeasible. *Note: the objective function will get worse as a result of the pivot.*

__o_ l. A basic solution of an LP is always feasible, but not all feasible solutions are basic. *Note: basic solutions may be either feasible or infeasible.*

__+_ m. In Phase One of the 2-Phase method, one should never pivot in the column of an artificial variable.

__+_ n. In a transportation problem if the total supply exceeds total demand, a "dummy" destination should be defined.

__o_ o. If the optimal value of a slack variable of a primal LP constraint is positive, then the optimal value of the dual variable for that same constraint must also be positive. *Note: according to complementary slackness theory, if a primal constraint is slack, the corresponding dual variable must be zero !*

(2.) ***Sensitivity Analysis in LP.***

Problem Statement: McNaughton Inc. produces two steak sauces, spicy Diablo and mild Red Baron. These sauces are both made by blending two ingredients A and B. A certain level of flexibility is permitted in the formulas for these products. Indeed, the restrictions are that:

i) Red Baron must contain no more than 75% of A.

ii) Diablo must contain no less than 25% of A and no less than 50% of B

Up to 40 quarts of A and 30 quarts of B could be purchased. McNaughton can sell as much of these sauces as it produces at a price per quart of $3.35 for Diablo and $2.85 for Red Baron. A and B cost $1.60 and $2.05 per quart, respectively. McNaughton wishes to maximize its net revenue from the sale of these sauces.

Define *D = quarts of Diablo to be produced*
R = quarts of Red Baron to be produced
AD= quarts of A used to make Diablo
AR = quarts of A used to make Red Baron
BD = quarts of B used to make Diablo
BR = quarts of B used to make Red Baron

The LINDO output for solving this problem follows:

```
MAX     3.35 D + 2.85 R - 1.6 AD - 1.6 AR - 2.05 BD - 2.05 BR
SUBJECT TO
      2) - D + AD + BD =    0
      3) - R + AR + BR =    0
      4)   AD + AR <=   40
      5)   BD + BR <=   30
      6) - 0.25 D + AD >=   0
      7) - 0.5 D + BD >=   0
      8) - 0.75 R + AR <=   0
END
       OBJECTIVE FUNCTION VALUE
 1)        99.0000000
```

VARIABLE	VALUE	REDUCED COST
D	50.000000	0.000000
R	20.000000	0.000000
AD	25.000000	0.000000
AR	15.000000	0.000000
BD	25.000000	0.000000
BR	5.000000	0.000000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	0.000000	-2.350000
3)	0.000000	-4.350000
4)	0.000000	0.750000
5)	0.000000	2.300001
6)	12.500000	0.000000
7)	0.000000	-1.999999
8)	0.000000	2.000000

RANGES IN WHICH THE BASIS IS UNCHANGED

OBJ COEFFICIENT RANGES

VARIABLE	CURRENT COEF	ALLOWABLE INCREASE	ALLOWABLE DECREASE
D	3.350000	0.750000	0.500000
R	2.850000	0.500000	0.375000
AD	-1.600000	1.500001	0.666666
AR	-1.600000	0.666666	0.500000

BD	-2.050000	1.500001	1.000000
BR	-2.050000	1.000000	1.500001

RIGHTHAND SIDE RANGES

ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
2	0.000000	10.000000	10.000000
3	0.000000	16.666668	3.333333
4	40.000000	50.000000	10.000000
5	30.000000	10.000000	16.666664
6	0.000000	12.500000	INFINITY
7	0.000000	6.250000	5.000000
8	0.000000	2.500000	12.500000

THE TABLEAU:

ROW	(BASIS)	D	R	AD	AR	BD	BR	SLK 4	SLK 5	SLK 6
1	ART	0.000	0.000	0.000	0.000	0.000	0.000	0.750	2.300	0.000
2	AD	0.000	0.000	1.000	0.000	0.000	0.000	-0.500	1.500	0.000
3	R	0.000	1.000	0.000	0.000	0.000	0.000	2.000	-2.000	0.000
4	AR	0.000	0.000	0.000	1.000	0.000	0.000	1.500	-1.500	0.000
5	BR	0.000	0.000	0.000	0.000	0.000	1.000	0.500	-0.500	0.000
6	SLK 6	0.000	0.000	0.000	0.000	0.000	0.000	-0.250	0.750	1.000
7	D	1.000	0.000	0.000	0.000	0.000	0.000	-1.000	3.000	0.000
8	BD	0.000	0.000	0.000	0.000	1.000	0.000	-0.500	1.500	0.000

ROW	SLK 7	SLK 8	RHS
1	2.000	2.000	99.000
2	3.000	2.000	25.000
3	-4.000	-4.000	20.000
4	-3.000	-2.000	15.000
5	-1.000	-2.000	5.000
6	2.000	1.000	12.500
7	4.000	4.000	50.000
8	1.000	2.000	25.000

_d_1. If the selling price of DIABLO sauce were to increase from $3.35 /quart to $4.50/quart, the number of quarts of DIABLO to be produced would

a. increase
b. decrease
c. remain the same
d. insufficient info. given
e. *NOTA*

Note: The proposed increase is above the "ALLOWABLE INCREASE" (0.75), and so the basis will change. While it is reasonable to suppose that the result of the basis change will be an increase in the quantity of DIABLO which is produced, it is not certain.

_a_2. The LP problem above has

a. exactly one optimal sol'n
b. a degenerate solution
c. multiple solutions
d. no optimal solution
e. insufficient info. given
f. *NOTA*

Note: Since there is no nonbasic variable with a zero reduced cost, including slack variables, the optimal solution is unique. Since no zero appears on the right-hand-side of the tableau, the solution is not degenerate.

_c_3. If an additional 5 quarts of ingredient B were available, McNaughton's profits would be *(choose nearest value)* :

a. $90
b. $100
c. $110
d. $120
e. insufficient info. given
f. *NOTA*

Note: The DUAL PRICE of row (5) is 2.3, and the ALLOWABLE INCREASE in the RHS is 10, the additional 5 quarts will increase the profit by 5(2.3) = 11.5 , i.e., from 99 to 110.5.

_b_4. If the variable "SLK 4" were increased, this would be equivalent to

a. increasing A availability
b. decreasing A availability
c. increasing B availability
d. decreasing B availability
e. *NOTA*

_b_5. If the variable "SLK 4" were decreased by 10 (i.e. from 0 to -10), the quantity of DIABLO produced would be *(choose nearest value)*

a. 30 quarts   c. 50 quarts   e. ≥70 quarts
b. 40 quarts   d. 60 quarts   f. *NOTA*

Note: According to the tableau, the substitution rate of SLK4 for the variable D is -1.000. Hence, the variable D will also decrease by 10, i.e. from its current value of 50 to 40.

_c_6. If a pivot were to be performed to enter the variable SLK4 into the basis, then according to the "minimum ratio test", the value of SLK4 in the resulting basic solution would be *(choose nearest value)*

a. 20   c. 10   e. ≤0.5
b. 15   d. 0.10   f. *NOTA*

Note: the ratios in rows (3), (4), and (5) are all 10.

_i_7. If the variable SLK4 were to enter the basis, then the variable leaving the basis is

a. A   c. AD   e. D   g. SLK6   i. more than one answer is possible
b. B   d. BD   f. R   h. any of the above   j. *NOTA*

Note: Since there is a "tie" in the minimum ratio test, the pivot could be performed in any one of rows (3), (4), or (5), which would remove either variables R, AR, or BR from the basis, respectively.

_b_8. If the variable SLK4 were to enter the basis, then the next tableau

a. indicates multiple optimal sol'ns   c. both of the above
b. is degenerate   d. *NOTA*

Note: whenever a tie is broken in the minimum ratio test, a zero will appear on the right-hand-side as a result of the pivot! (In this case, two zeroes will appear.) You were not expected to actually perform the pivot, which would be required in order to determine if the resulting tableau was optimal, with multiple optima indicated. If you were to actually perform the pivot, the resulting tableau would not be optimal, of course, since it is optimal for SLK4 to remain nonbasic!

_a_9. The dual of the LP above has an objective function which is to be

a. minimized   c. both of the above
b. maximized   d. *NOTA*

_c_10. The dual of the LP above has an optimal value which is *(choose nearest value)*

a. 0   c. 100   e. insufficient infomation given
b. 50   d. ≥150   f. *NOTA*

Note: LP duality theory says that the optimal values of the primal and dual LPs are the same, if either one of them has a feasible optimum.

tststs PART TWO tststs

(3.) ***Simplex Method.*** Classify each simplex tableau below by writing "X" in the appropriate (one or more) columns, using the following classifications:

- Is the current solution feasible or not?
- Is the current solution degenerate or not?
- Is there an indication that the LP has an unbounded objective function?
- Is the current solution optimal?
- If the current solution is optimal, are there other optima?

In the tableaus which are feasible but not optimal, circle at least one valid pivot element to improve the objective. *Take careful note of whether the LP is being **min**imized or **max**imized! Note also that (-z), rather than z, appears in the first column (i.e., corresponding to the approach used in my notes instead of that in the text by Winston).*

	-Z	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	RHS	Feasible?	Degener.?	UNBounded?	Optimal?	Multi-Opt?
MIN	1	2	0	4	-3	-2	0	1	0	-10	(X)	(_)	(X)	(_)	(_)
	0	0	0	2	-4	0	0	-1	1	3					
	0	-3	1	0	-1	2	0	2	0	6					
	0	2	0	3	0	**(5)**	1	1	0	2					

	-Z	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	RHS	Feasible?	Degener.?	UNBounded?	Optimal?	Multi-Opt?
MAX	1	-2	0	-4	-2	-3	0	1	0	-10	(X)	(_)	(_)	(_)	(_)
	0	0	0	2	1	0	0	-1	1	3					
	0	-3	1	0	-1	2	0	2	0	6					
	0	2	0	3	0	5	1	**(1)**	0	2					

	-Z	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	RHS	Feasible?	Degener.?	UNBounded?	Optimal?	Multi-Opt?
MIN	1	0	0	4	2	3	0	1	0	-10	(X)	(X)	(_)	(X)	(X)
	0	0	0	2	1	0	0	-1	1	3					
	0	-3	1	0	-1	2	0	2	0	6					
	0	2	0	3	0	5	1	1	0	0					

	-Z	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	RHS	Feasible?	Degener.?	UNBounded?	Optimal?	Multi-Opt?
MAX	1	-2	0	-4	-2	-3	0	1	0	-10	(X)	(X)	(_)	(_)	(_)
	0	0	0	2	1	0	0	-1	1	0					
	0	-3	1	0	-1	2	0	2	0	6					
	0	2	0	3	0	5	1	**(1)**	0	2					

	-Z	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	RHS	Feasible?	Degener.?	UNBounded?	Optimal?	Multi-Opt?
MAX	1	2	0	4	-2	-3	0	1	0	-10	(_)	(_)	(_)	(_)	(_)
	0	0	0	2	1	0	0	-1	1	-3					
	0	-3	1	0	-1	2	0	2	0	6					
	0	2	0	3	0	5	1	1	0	2					

4. **LINEAR PROGRAMMING DUALITY**: Consider the following LP:

Maximize $2X_1 - 13X_2 - 3X_3 - 2X_4 - 5X_5$

subject to

$X_1 - X_2 - 4X_4 - X_5 = 5$

$X_1 - 7X_4 - 2X_5 \geq -1$

$5X_2 + X_3 + X_4 + 2X_5 \leq 5$

$3X_2 + X_3 - X_4 + X_5 \geq 2$

$X_j \geq 0$ for all j=1, 2, 3; $X_4 \leq 0$; X_5 unrestricted in sign

At the primal point X=(6,0,1,0,1),

objective function = -4

left-hand-side of 1st constraint is 5 = *5 (right-hand-side)*

left-hand-side of 2nd constraint is 4 *> -1 => surplus is positive!*

left-hand-side of 3rdconstraint is 3 *< 0 => slack is positive!*

left-hand-side of 4th constraint is 2 = *2 (right-hand-side)*

a. Is this solution feasible? YES

b. Is this solution basic? NO *Note: The number of basic variables, excluding the objective (-z), must be 4 (the number of linear constraints). Since 3 X's are positive and 2 slack variables (in 2nd and 3rd constraints) are positive, we have 5 positive variables!*

c. Is this solution degenerate? NO

d. Complete the following properties of the dual problem of this LP:

Number of dual variables: __4_

Number of dual constraints (not including nonnegativity): __5__

Type of optimization Minimize

e. Write out in full a dual problem of the LP above, denoting your dual variables by Y_1, Y_2, etc..

Minimize $5Y_1 - Y_2 + 5Y_3 + 2Y_4$

subject to

$Y_1 + Y_2 + 3Y_4 \geq 2$

$-Y_1 + 5Y_3 + Y_4 \geq -13$

$Y_3 + Y_4 \geq -3$

$-4Y_1 - 7Y_2 + Y_3 - Y_4 \leq -2$

$-Y_1 - 2Y_2 + 2Y_3 + Y_4 = -5$

Y_1 unrestricted in sign, $Y_2 \leq 0$, $Y_3 \geq 0$, $Y_4 \leq 0$

f. IF X=(6,0,1,0,1) is optimal in the primal problem, then which **dual** variables (including slack or surplus variables) must be **zero** in the dual optimal solution, according to the complementary slackness conditions for this primal-dual pair of problems?

Note: the original intention was that X=(6,0,1,0,1) would be the basic optimal solution, but as discussed in (b) above, it is not basic. If it were nonbasic but optimal, then there would be (at least) two optimal basic solutions, with a corresponding degenerate dual solution (with two different optimal bases).

The original intended answer (which assumed that the solution was basic) is as follows:

- Since the 2nd and 3rd constraint are slack, then the corresponding variables Y_2 and Y_3 must be zero!
- Since X_1, X_3, and X_5 are positive, the slack or surplus in the corresponding dual constraints must be zero!

5. **Transportation Problem:** Consider the transportation problem with the tableau below:

destinations

sources	1	2	3	supply
1	5 [9]	[5]	[4]	5
2	[7]	[6]	[12]	18
3	[10]	[9]	[3]	7
demand	10	5	15	

a. If the ordinary simplex tableau were to be written for this problem, how many rows (including the objective) will it have? __7__
How many variables (excluding the objective value -z) will it have? __9___

b. Is this transportation problem "balanced?" __yes_ *(since sum of supplies = sum of demands)*

c. How many basic variables will this problem have? __5_ = *m+n-1*

d. An initial basic feasible solution is found using the "Northwest Corner Method"; complete the computation of this solution and write the values of the variables in the tableau.

destinations

sources	1	2	3	supply
1	5 [9]	[5]	[4]	5
2	5 [7]	5 [6]	8 [12]	18
3	[10]	[9]	7 [3]	7
demand	10	5	15	

e. If U_1 (the dual variable for the first source) is equal to 0, what is the value of

U_2 (the dual variable for the second source)? __-2__ $C_{21} = 7$, $V_1 = 9$ & $U_2 + V_1 = C_{21}$ $U_2 = -2$

V_1 (the dual variable for the first destination)? __9___ $U_1 = 0$ & $U_1 + V_1 = C_{11}$ $V_1 = 9$

V_2 (the dual variable for the second destination)? __8__ $C_{22} = 6$, $U_2 = -2$ & $U_2 + V_2 = C_{22}$ $V_2 = 8$

Note that after setting $U_1=0$, you should compute these dual variables in the order V_1, U_2, and V_2.

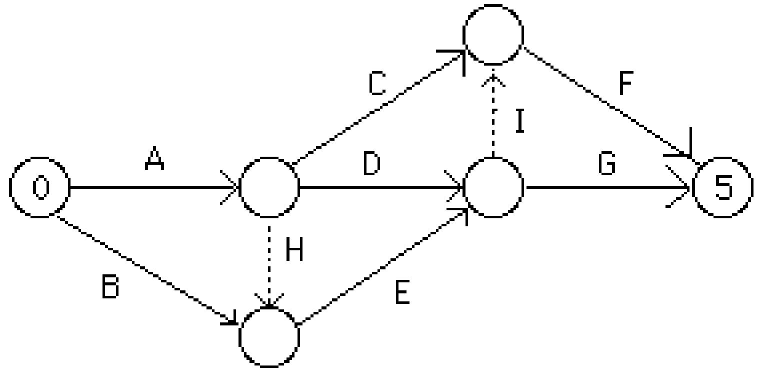
f. What is the reduced cost of the variable X_{12}? __-3__ Note: $C_{12} - (U_1 + V_2) = 5 - (0+8) = -3$

g. Will increasing X_{12} improve the objective function? _Yes_ *Note: each unit increase of X_{12} will lower the cost by $3.*

h. Regardless of whether the answer to (f) is "yes" or "no", what variable must leave the basis if X_{12} enters?
Answer: Either X_{11} or X_{22} *Note: Both X11 & X22 will become zero simultaneously as X12 is increased. The choice of the variable to leave the basis is arbitrary.*

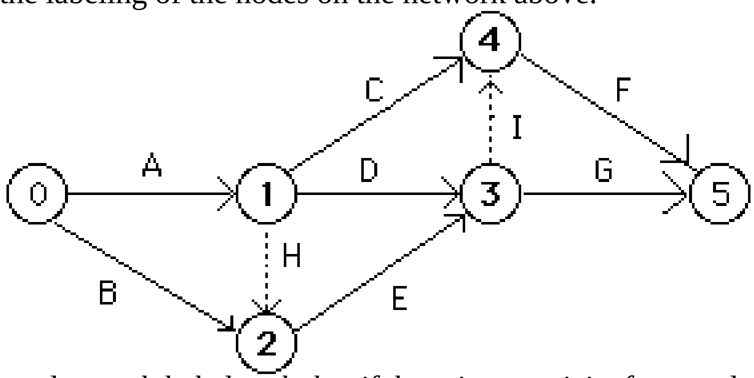
i. What will be the value of X_{12} if it is entered into the solution as in (h)? ___5___

6. ***Project Scheduling.*** Consider the project with the A-O-A (activity-on-arrow) network given below.



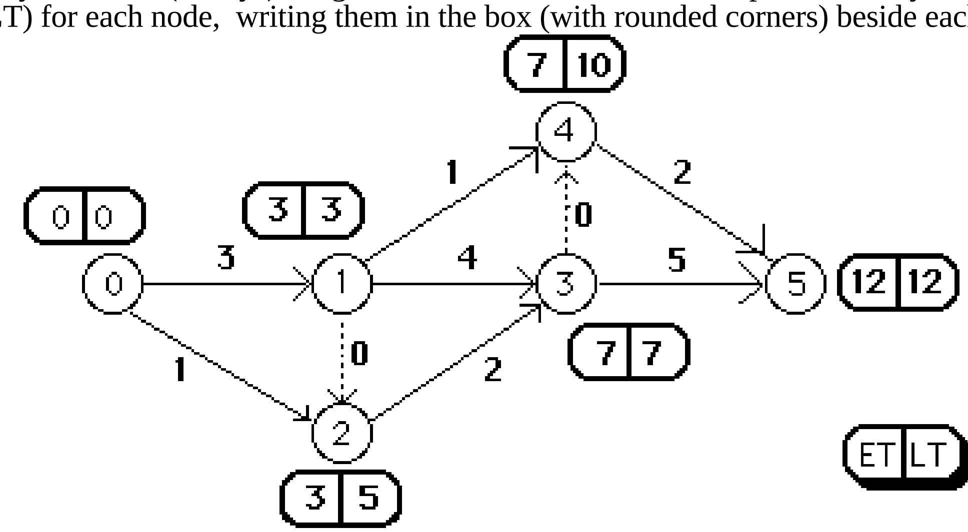
a. How many activities (i.e., tasks), not including "dummies", are required to complete this project? __7__

b. Complete the labeling of the nodes on the network above.



Note: The nodes are labeled such that if there is an activity from node i to node j, then i<j.

c. The activity durations (in days) are given below on the arrows. Compute the Early Times (ET) and Late Times (LT) for each node, writing them in the box (with rounded corners) beside each node.

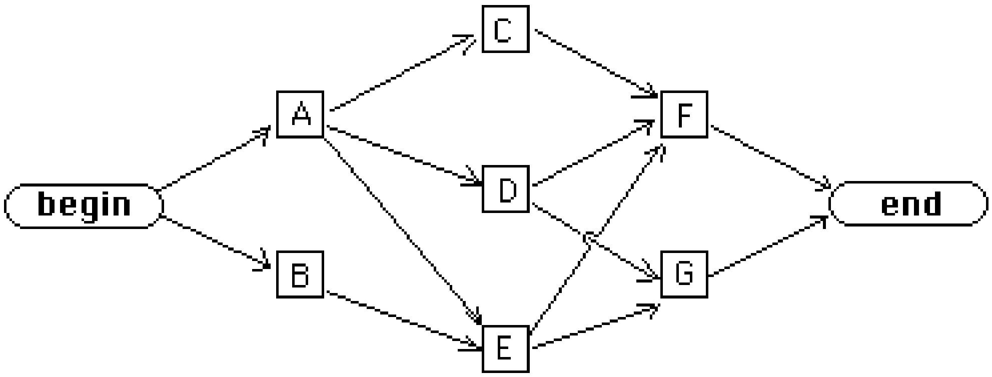


d. Find the slack ("total float") for activity B. __4_days. Note: Early start (ES) time for activity B is 0 and late finish (LT) time is 5. Therefore, late start (LS) time is 1 day before LF, i.e., LS=4. The total float is therefore EF - ES = 4 - 0 = 4.

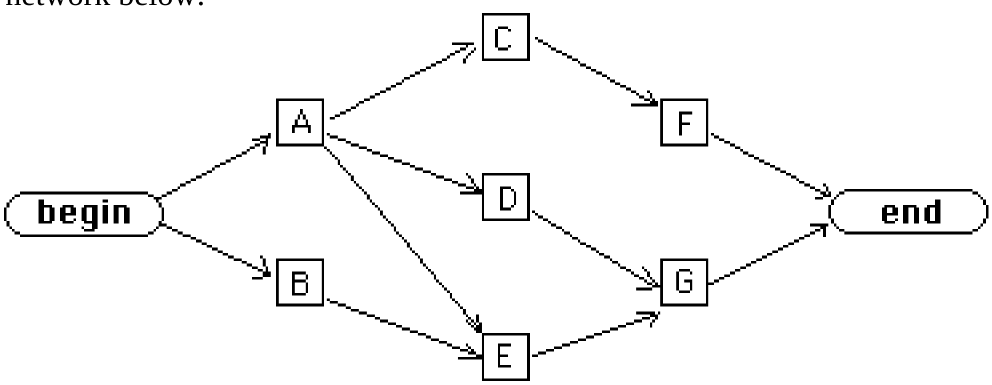
e. Which activities are critical? **(A)** B C **(D)** E F **(D)** H I

f. What is the earliest completion time for the project? __12_days

g. Complete the A-O-N (activity-on-node) network below for this same project.



h. Suppose that the arrow labelled "I" in the original AOA network is deleted. Indicate the resulting A-O-N network below:



7. Decision Analysis. We have $1000 to invest in one of the following: Gold, Stock, or Money Market. The value of the $1000 investment a year from now depends upon the unknown state of the economy in the intervening year. The value of the investment one year from now is given by the table:

Investment	Weak	Moderate	Strong	Min	Max
Money market	$1100	$1100	$1100	1100	1100
Stock	$1000	$1100	$1200	1000	1200
Gold	$1600	$300	$1400	300	1600

a. What is the optimal investment decision if your criterion is "maximin"? _Money market (maximin=1100)
What is the optimal investment decision if your criterion is "maximax"? _Gold (maximax = 1600)

b. Complete the regret table:

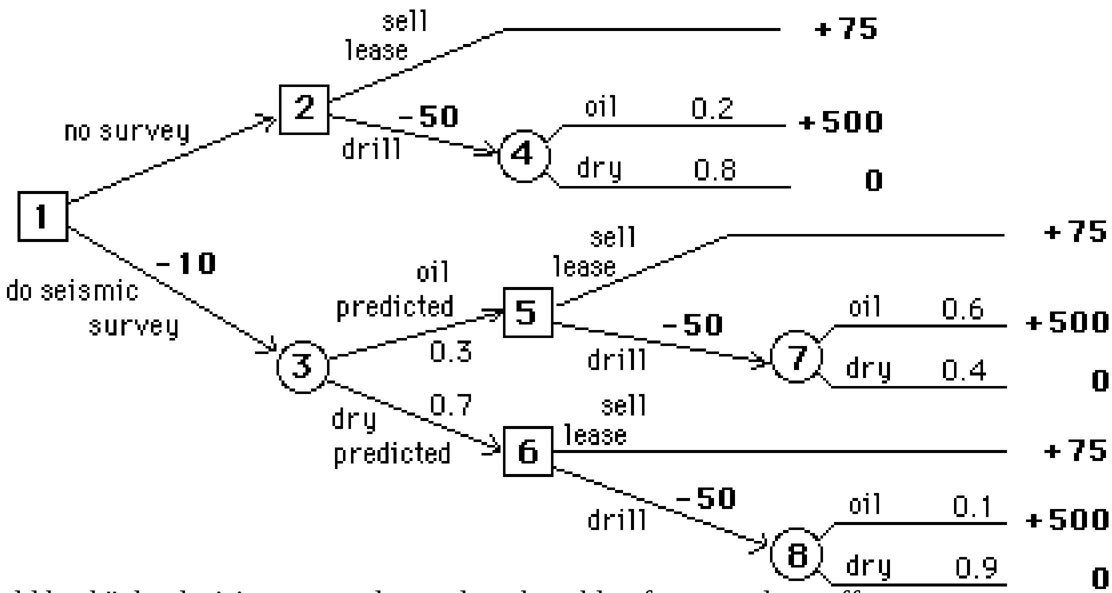
Investment	Weak	Moderate	Strong	Max
Money market	_500_	__0__	_300_	500
Stock	_600_	__0__	_200_	600
Gold	__0__	_800_	__0__	800

c. What is the optimal investment decision if your criterion is "minimax regret"? _Money market _(minimax regret = 500)

Suppose that you own a lease on the oil rights of a piece of land. You have the options of

- selling the lease for $75,000
- drilling for oil yourself, which costs you an investment of $50,000

If you choose to drill, the estimated probability of finding oil is 20%, in which case the "payoff" is $500,000. If the oil well is "dry", there is no payoff, of course.
Before you make the above decision, you have the option of hiring a geologist to do a seismic survey for $10,000. The geologist will predict either that there is oil or that the well will be dry. If he predicts oil, he has been right 60% of the time. When he predicts a dry well, he is right 90% of the time.
The probability that the geologist will predict oil is 30%.
These values have been inserted in a decision tree shown below, with costs and payoffs expressed in thousands of dollars. *Note that the costs of the survey and of the drilling are indicated on the decision branches, and not included in the final payoff at the right!*



"Fold back" the decision tree and complete the table of expected payoffs:

node	E[payoff]	node	E[payoff]	node	E[payoff]
1	_117.5_	4	__100_	7	__300_
2	__75___	5	__250_	8	___50_
3	_127.5_	6	___75_		

d. Should you hire the geologist to perform the seismic survey? __YES__

e. What is the expected value of the geologist's survey? __52.5__ (thousands of dollars)
Note: EVSI = EVWI - EVWOI = 127.50 - 75 = 52.5.