56:171 Operations Research
Midterm Exam Solutions
October 22, 1993

(A.) ***True/False:*** Indicate by "+" ="true" or "o" ="false" :

False 1. A "dummy" activity in CPM has duration zero and cannot be on the critical path.

True 2. In PERT, the total completion time of the project is assumed to be a random variable with a normal distribution.

True 3. A "pivot" in a nonbasic column of a tableau will make it a basic column.

True 4. The earliest completion time of a project could be computed by formulating an LP problem and solving it with the simplex method.

False 5. In the two-phase simplex method, Phase One computes the optimal dual variables, followed by Phase Two in which the optimal primal variables are computed. *(Phase One eliminates the artificial variables, while Phase Two computes the optimal primal variables.)*

False 6. "Crashing" a project is a procedure for allowing additional time for critical activities. *(In Crashing a project, the completion time of the project is reduced by reducing the time of activities.)*

True 7. In CPM, the "backward pass" is used to determine the latest time (LT) for each event (node).

True 8. Considered as an LP problem, every basic feasible solution of an assignment problem is degenerate.

True 9. The "minimum ratio test" is used to determine the pivot row in the simplex method.

False 10. During any iteration of the simplex method, if x_j is the variable entering the basis, the improvement in the cost function resulting from the pivot is the value of the reduced cost. *(The reduced cost gives the improvement per unit of the variable entering the basis, and so the improvement is the reduced cost multiplied by the result of the minimum ratio test.)*

False 11. In applying the Hungarian method to an assignment problem, the number of iterations required may depend upon the degree of degeneracy of the problem. *(Every basic solution of an assignment problem has the same degree of degeneracy, i.e., the same number of basic variables equal to zero, namely n-1.)*

False 12. Before you enter an LP formulation into LINDO, you must first convert all inequalities to equations. *(This is necessary for the simplex method, but it is not necessary for you to do, as LINDO will do this for you.)*

True 13. The A-O-N project network does not require any "dummy" activities, except for the "begin" and "end" activities.

True 14. The revised simplex method, unlike the ordinary simplex method which pivots in the original tableau, leaves the original tableau unchanged.

True 15. If an artificial variable is nonzero in the optimal solution of an LP problem, then the problem has no feasible solution.

True 16. At the end of the first phase of the two-phase simplex method, the phase-one objective function must be zero if the LP is feasible.

False 17. If a zero appears on the right-hand-side of row i of an LP tableau, then at the next iteration you *must* pivot in row i. *(Only if there is a positive substitution rate in this row of the pivot column is it necessary to pivot in row i (or another row satisfying the same conditions.))*

False 18. In the Hungarian method, no further reduction of the cost matrix is necessary if the number of lines required to cover the zeroes is less than the number of rows in the cost matrix. *(In the*

Hungarian method, one stops when the minimum number of lines required to cover the zeroes is equal to the number of rows.)

False 19. In a transportation problem, if the total demand exceeds total supply, a "dummy" destination should be defined. *(If demand exceeds supply, then the problem is infeasible. If a shortage penalty is given, one could define a dummy destination, with a supply equal to the amount of the shortfall.)*

True 20. In a transportation problem, the number of basic (primal) variables is less than the number of dual variables. *(The number of basic primal variables is m+n-1, while the number of dual variables is m+n.)*

True 21. In a transportation problem, if the current dual variables U_2=3 and V_4=1, and C_{24}=2, then the current basic solution cannot be optimal.

True 22. In a transportation problem, if the current dual variables U_2=3 and V_4=1, and C_{24}=5, then X_{24} cannot be basic.

True 23. If the "float" of an activity of a project is positive, then the activity cannot be "critical" in the schedule.

True 24. All tasks on the critical path of a project schedule have their latest finish time equal to their earliest finish time.

False 25. If a zero appears on the right-hand-side of row i of an LP tableau, then at the next iteration you *cannot* pivot in row i. *(If there is a positive value in row i of the pivot column, then you must pivot in row i.)*

True 26. When you enter an LP formulation into LINDO, you must manipulate your equality constraints so that all variables appear on the left, and all constants on the right of the "=".

False 27. A transportation problem is called "balanced" if the number of supply points equals the number of demand points. *(It is "balanced" if the total amount of the supply equals the total amount of the demand, regardless of the number of supply points (m) and demand points (n).)*

False 28. When maximizing in the simplex method, the value of the objective function cannot improve at the next pivot if the current tableau is degenerate. *(If it is not necessary to pivot in a row with zero on the right-hand-side (because the substitution rate in this row is negative or zero), and the relative profit is strictly positive, then there will be an increase in the objective function.)*

True 29. When minimizing in the simplex method, the cost may be improved by selecting any column having a negative reduced cost as the pivot column.

False 30. A basic solution of an LP is always feasible, but not all feasible solutions are basic. *(See Problem B of this exam for counterexamples of both statements.)*

False 31. The optimal value of a primal minimization LP problem is less than or equal to the objective value of every dual feasible solution. *(The optimal value of the minimization problem is greater than or equal to the objective value of every dual feasible solution (and equal to that of the dual optimal solution.))*

True 32. The optimal values of the primal and dual LP problems, if they exist, must be equal.

False 33. If a primal minimization LP problem has a cost which is unbounded below, then the dual maximization problem has an objective which is unbounded above. *(In this situation, the dual problem cannot be feasible.)*

False 34. If X_{ij}=0 in the transportation problem, then dual variables U and V *must* satisfy $C_{ij}=U_i+V_j$. *(Only if X_{ij} is basic must $C_{ij}=U_i+V_j$. So it is possible, in case of degeneracy, that the equation will hold, but it is certainly not necessary.)*

True 35. In project scheduling, the problem of finding the earliest completion time for the project can be stated as an LP, with a dual LP which will find the length of the longest path from beginning to ending of the project.

<u>False</u> 36. The reduced cost of a slack variable in row i is the simplex multiplier $ð_i$ for that row (if ˉz is used as the basic variable in the objective row). *(The reduced cost of the slack variable must equal the cost of the slack variable (i.e., zero) minus the simplex multiplier vector times the slack column, which will be the negative of the simplex multiplier.)*

<u>True</u> 37. At the completion of the revised simplex method applied to an LP, the simplex multipliers give the optimal solution to the dual of the LP.

<u>True</u> 38. In the revised simplex method, before entering variable X_j into the basis, the substitution rates (necessary for the minimum ratio test) are computed by multiplying the basis inverse matrix times the original column of constraint coefficients for X_j.

<u>False</u> 39. One advantage of the revised simplex method is that it does not require the use of artificial variables. *(In the revised simplex method, as in the ordinary simplex method, you must begin with a basic feasible solution. This may require performing Phase One (or Big-M method), using artificial variables.)*

<u>True</u> 40. If you change the objective coefficients of an LP which you solved yesterday, you can use yesterday's optimal solution as the starting basic feasible solution to solve the new problem today.

<u>True</u> 41. If the simplex method is applied to the transportation problem, all of the "substitution rates" which are computed for the optimal solution will be either +1, -1, or zero.

<u>False</u> 42. In the LP formulation of the project scheduling problem, the constraints include $Y_A - Y_B$ d_A if activity A must precede activity B, where d_A is the given duration of activity A. *(The constraint should be* $Y_B - Y_A$ d_A *, or* Y_B $Y_A + d_A$ *, i.e., the start time for B must be greater than or equal to (i.e., no earlier than) the completion time of A.)*

<u>True</u> 43. Bayes' Rule can be used for revising one's estimates of the defective rate of a manufacturing process after one has inspected a sample of items obtained from the process.

<u>False</u> 44. If you increase the right-hand-side of a "less-than-or-equal" constraint in a minimization LP, the optimal objective value will either increase or stay the same. *(Increasing the right-hand-side of a "less-than-or-equal" constraint makes the constraint less restrictive, and so the objective function might improve, which in the case of a minimization problem, means decrease.)*

<u>False</u> 45. The "reduced cost" in LP provides an estimate of the change in the objective value when a right-hand-side of a constraint changes. *(The dual variable provides an estimate of the rate of change of the objective value when the right-hand-side changes.)*

<u>False</u> 46. The transportation problem is a special case of an assignment problem. *(The assignment problem is a special case of a transportation problem, but not every transportation problem is an assignment problem.)*

(B.) ***Multiple Choice:*** Write the appropriate letter (a, b, c, d, or e) : (*NOTA* =<u>N</u>one <u>o</u>f <u>t</u>he <u>a</u>bove).

<u>*NOTA*</u> 1. If, in the optimal *primal* solution of an LP problem (min cx st $Ax \leq b$, $x \geq 0$), there is zero slack in constraint #1, then in the optimal dual solution,

(a) dual variable #1 must be zero (c) slack variable for dual constraint #1 must be zero
(b) dual variable #1 must be positive (d) dual constraint #1 must be slack (e) *NOTA*

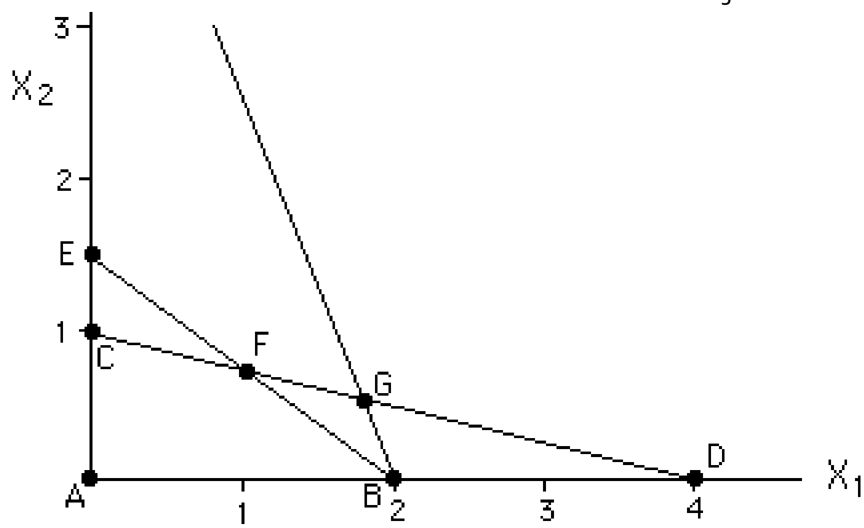
(dual variable #1 might be positive, but it is possible that both the slack in the primal constraint and the corresponding dual variable are zero, in which case the solution is degenerate.)

2. If, in the optimal *dual* solution of an LP problem (min cx st Ax ≤ b, x ≥ 0), variable #2 is positive, then in the optimal primal solution, slack variable for constraint #2 must be zero *(This is a consequence of the Complementary Slackness Theorem.)*
3. If you make a mistake in choosing the pivot row in the simplex method, the solution in the next tableau will be nonfeasible
4. If you make a mistake in choosing the pivot column in the simplex method, the solution in the next tableau will have a worse objective value
5. If there is a tie in the "minimum-ratio test" of the simplex method, the solution in the next tableau will be degenerate

The problems (6)-(10) below refer to the following LP:

(with inequalities converted to equations:)

Minimize $8X_1 + 4X_2$	Minimize $8X_1 + 4X_2$
subject to $3X_1 + 4X_2 \quad 6$	subject to $3X_1 + 4X_2 - X_3 = 6$
$5X_1 + 2X_2 \quad 10$	$5X_1 + 2X_2 + X_4 = 10$
$X_1 + 4X_2 \quad 4$	$X_1 + 4X_2 + X_5 = 4$
$X_1 \quad 0,\ X_2 \quad 0$	$X_j \quad 0,\ j=1,2,3,4,5$



6 .The feasible region includes points **B, F, & G**

7. At point F, the basic variables include the variables **X_1 & X_4** ***(in addition to X_2)***

8. Which point is degenerate in the primal problem? **point B**

(At point B, X_2, X_3, and X_4 are zero, and the only positive variables are X_1 and X5. Since there are three constraints, this means that one of the basic variables must be zero.)

9. If point F is optimal, then which dual variables must be zero, according to the Complementary Slackness Theorem? **Y_2 only**

(At point F, the slack variable for the second constraint, X_4, is positive, and so the dual variable for this constraint must be zero.)

10. For each alternative pair in parentheses, check the appropriate choice to obtain the dual LP of the above primal problem (with the inequality constraints):

Max $6Y_1 + 10Y_2 + 4Y_3$

subject to $3Y_1 + 5Y_2 + Y_3 \quad 8$

$4Y_1 + 2Y_2 + 4Y_3 \quad 4$

$Y_1 \quad 0,\ Y_2 \quad 0,\ Y_3 \quad 0$

(C.) ***Sensitivity Analysis in LP.***

1. How many yards of Beslite should be manufactured? <u>5600 yards</u>
2. How much of the available diurethane will be used?
   a. 6900 pounds c. 13100 pounds e. NSI
   b. 1600 pounds d. 400 pounds
3. How much of the available diurethane will be unused? <u>13100 pounds</u>

<u>NO</u> 4. Suppose that the company can purchase 2000 pounds of additional polyamine for $2.50 per pound. Should they make the purchase?

<u>YES</u> 5. If the profit contribution from Beslite were to decrease to $12/yard, will the optimal solution change?

<u>NO</u> 6. If the profit contribution from Ankelor were to increase to $15/yard, will the optimal solution change?

<u>YES</u> 7. Suppose that the company could deliver 1000 yards less than the contracted amount of Ankalor by paying a penalty of $5/yard shortage. Should they do so?

8. Regardless of your answer in (7), suppose that they <u>do</u> deliver 1000 yards less Ankalor. This is equivalent to <u>decreasing the slack in row 6 by 1000</u>

9. If the company delivers 1000 yards less of Ankalor, how much Beslite should they deliver? <u>6400 yards</u>

10. How will the decision to deliver 1000 yards less Ankalor change the quantity of diurethane used during the next planning period? <u>decrease by 1700 pounds</u>