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56:271 Nonlinear Programming
Quiz #2 --- September 9, 2003

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Indicate whether *true* (+) or *false* (o).

___ 1. The Hessian matrix of a function $f(x_1,x_2)$ is denoted by $\nabla^2 f(x_1,x_2)$.

___ 2. If f is a quadratic function, then $\nabla^2 f(x_1,x_2)$ is a constant matrix.

___ 3. If f is a quadratic function, then f is convex.

___ 4. The first row of the Jacobian matrix of the system of equations
$g_1(x_1, \ldots x_n)=0, g_2(x_1, \ldots x_n)=0, \ldots g_n(x_1, \ldots x_n)=0,$
is $\nabla g_1(x_1, \ldots x_n)$, the gradient of g_1.

___ 5. The matrix $\begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}$ is *positive definite*.

___ 6. The matrix $\begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}$ is *positive semi-definite*.

___ 7. The Jacobian matrix of any system of equations is symmetric.

___ 8. A function $f(x)$ of one variable is convex if its graph lies on or above any tangent line to that graph.

___ 9. The function $f(x) = 2x_1^2 + x_1x_2 - x_2^2 + 5x_1 + 3$ is a *quadratic form.*

Consider the nonlinear system of equations

$$\begin{cases} x^2 + 3y^2 = 7 \\ 2xy + x = 6 \end{cases} \quad \text{or} \quad \begin{cases} g_1(x,y) = 0 \\ g_2(x,y) = 0 \end{cases} \quad \text{where} \quad \begin{cases} g_1(x,y) = x^2 + 3y^2 - 7 \\ g_2(x,y) = 2xy + x - 6 \end{cases}$$

and $\nabla g_1(x,y) = \begin{bmatrix} 2x \\ 6y \end{bmatrix}, \nabla g_2(x,y) = \begin{bmatrix} 2y+1 \\ 2x \end{bmatrix}$

10. The *Jacobian* matrix of this system is

$$J(x,y) = \begin{bmatrix} \_\_\_\_ & \_\_\_\_ \\ \_\_\_\_ & \_\_\_\_ \end{bmatrix}$$

Suppose we begin the Newton-Raphson algorithm at the point $(x^0, y^0) = \left(\tfrac{1}{2}, \tfrac{1}{2}\right)$.

11. The approximate solution after one iteration is $(x^1, y^1) = (x^0, y^0) + (\delta_x, \delta_y)$ where δ is found by computing

$$\begin{bmatrix} \delta_x \\ \delta_y \end{bmatrix} = -\begin{bmatrix} \_\_\_\_ & \_\_\_\_ \\ \_\_\_\_ & \_\_\_\_ \end{bmatrix}^{-1} \begin{bmatrix} \_\_\_\_ \\ \_\_\_\_ \end{bmatrix} = \begin{bmatrix} 1.8 \\ 1.4 \end{bmatrix}$$