56:271 Nonlinear Programming
Quiz 10 – Fall 2003

1. Two products X and Y can be assembled from two components A & B with availabilities b_A & b_B, respectively. The product mix LP model is

$$\begin{aligned} \max \quad & 250x + 185y \\ s.t. \quad & 2x + 3y \geq b_A \\ & x + 2y \geq b_B \\ & x, y \geq 0 \end{aligned}$$

For example, one unit of X requires 2 units of component A and one of component B. However, the availabilities b_A & b_B are random with cumulative distribution functions F_A & F_B, respectively. For example, $F_A(t) = P\{b_A \leq t\}$.

What (nonlinear) constraint would you use so as to be 80% confident that the solution *(x,y)* will be feasible, i.e., so that sufficient components are available to assemble the specified X and Y?

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2. Consider again the GP model for designing the concentric cylinders:

$$\begin{aligned} Min \quad & 4\pi r_1^2 h + 2\pi r_2 h + 2\pi r_2^2 \\ s.t. \quad & 1000 r_2^{-2} h^{-1} + r_1^2 r_2^{-2} \leq 1 \\ & 50 r_1^{-1} h^{-1} \leq 1 \\ & r_1, r_2, h > 0 \end{aligned}$$

where r_1 & r_2 are the radii of the outer & inner cylinders, respectively, and h their height.
This can be reformulated as a *separable convex* minimization problem

$$\begin{aligned} Min \quad & 4\pi e^{z_1} + 2\pi e^{z_2} + 2\pi e^{z_3} \\ s.t. \quad & 1000 e^{z_4} + e^{z_5} \leq 1 \\ & 50 e^{z_6} \leq 1 \end{aligned}$$

$$\begin{cases} z_1 = \_\_\_u_1 + \_\_\_u_2 + \_\_\_u_3 \\ z_2 = \_\_\_u_1 + \_\_\_u_2 + \_\_\_u_3 \\ z_3 = \_\_\_u_1 + \_\_\_u_2 + \_\_\_u_3 \\ z^4 = \_\_\_u_1 + \_\_\_u_2 + \_\_\_u_3 \\ z^5 = \_\_\_u_1 + \_\_\_u_2 + \_\_\_u_3 \\ z^6 = \_\_\_u_1 + \_\_\_u_2 + \_\_\_u_3 \end{cases}$$

a. Complete the coefficients of the equations above.

b. What sign restrictions, if any, should be placed on z_i? ______________

c. What sign restrictions, if any, should be placed on u_j? ______________

d. What is the relationship between

u_1 and r_1? ______________