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56:271 Nonlinear Programming
Midterm Exam -- October 21, 1988

1. (26 points) Indicate whether each statement is true or false . If false , explain why (or give a counter-example).

___ a. The Kuhn-Tucker conditions are necessary conditions for optimality.
___ b. The steepest descent algorithm requires the calculation of partial derivatives.
___ c. "GRG" means "Gradient Restriction Generation"
___ d. A linear function is neither convex nor concave.
___ e. ^{2}f is the Hessian matrix of the function f.
___ f. The Jacobian matrix of the equation f(x) = 0 is called the Hessian matrix.
___ g. The "independent" variables of the GRG algorithm are essentially the same as the "basic" variables of the simplex LP algorithm.
___ h. A twice-differentiable function is convex if and only if its Hessian matrix is nonnegative everywhere.
___ i. The function e^{ax} is convex for all values of a.
___ j. All quadratic functions are convex.
___ k. The function f(x,y) = xy is a concave function of x and y.
___ l. A positive definite matrix always has all positive elements, although a matrix whose elements are positive need not be positive definite.
___ m. The Lagrangian dual of a convex quadratic programming problem is a quadratic programming problem.
___ n. Powell's algorithm does not require the computation of partial derivatives.
___ o. The product of two convex functions is convex.
___ p. In the "Golden Section Search" method, one-third of the interval of uncertainty is eliminated at a typical iteration.
___ q. A function which is not convex is called concave.
___ r. "Quasi-Newton" search methods for unconstrained minimization require the computation of second partial derivatives.
___ s. The sum of two convex functions is convex.
___ t. The GRG algorithm requires that dependent variables not be at either their lower or upper bounds.
___ u. If the current x lies on the boundary of the feasible region, the gradient projection method computes a search direction by projecting the steepest descent method onto the boundary.
___ v. The gradient projection method computes Lagrange multipliers at each iteration, and stops when they have the appropriate sign.
___ w. The Fletcher-Powell method is also known as the "Conjugate Gradient" method.
___ x. The Fletcher-Reeves method is also known as a "Quasi-Newton" method.
___ y. If f(x) is quadratic, the minimum of f(x) may be found by performing one-dimensional searches in each of 2 conjugate directions.

___ z. Any two orthogonal directions are conjugate with respect to the identity matrix.

2. (20 points) Consider the problem

$$\text{Minimize } f(x) = (x_1 - x_2)^2 + x_2$$

$$\text{subject to} \quad \begin{aligned} x_1 + x_2 &\geq 1 \\ x_1 - 2x_2 &\leq 3 \\ x_1 \geq 0,\ x_2 &\geq 0 \end{aligned}$$

(a.) Write down Kuhn-Tucker conditions for the optimal solution of this problem. How many complementary slackness conditions are there?

(b.) Are the Kuhn-Tucker conditions satisfied at the point $x^o=(3,0)$?

(c.) Illustrate your answer in (b) by sketching the feasible region, the gradients of the tight constraints at x^o, and the steepest descent direction. Explain the relationship between your sketch and the K-T conditions.

(d.) Compute the Hessian matrix of the objective, and test whether it is positive semidefinite.

(e.) Are the Kuhn-Tucker conditions necessarily satisfied by any optimal solution of this problem?

(f.) Discuss a "constraint qualification" for this problem. Is it satisfied? What is the importance of a "constraint qualification"?

(g.) Is any point which satisfies the Kuhn-Tucker conditions which you have written necessarily optimal?

(h.) Explain (briefly) how complementary pivoting may be used to find a solution of the Kuhn-Tucker conditions.

(i.) Suppose that you were to solve the above problem using the Feasible Direction Algorithm, starting at x^o given above. Let d^o represent the search direction. What inequality or inequalities must be satisfied by d^o if it is to be feasible ? if it is to be a direction of improvement ? Write down an LP which will find (if one exists) a feasible direction of improvement.

3. (9 points) Consider the function $f(x) = x \log x$

a. Is f a convex function? a concave function?

b. Suppose that I wish to find x such that $f(x)=1$. Describe an iterative procedure to do this, and illustrate one step, starting at $x^o = 1$.

c. Suppose that I wish to find x (>0) minimizing the function f. Describe an iterative procedure to do this, and illustrate one step, starting at $x^o = 1$.