56:271 – Nonlinear Pgmg.
Midterm Exam: 10/12/87

<u>Select any 3 of the 5 problems below:</u>

(1.) Suppose that $\underline{x}$ is a stationary point of the function f, i.e., at $x=\underline{x}$, $f(\underline{x}) = 0$. What can you say about $\underline{x}$ if

(a.) f is a convex function?
(b.) f is a concave function?
(c.) $^2f(\underline{x})$ is indefinite?
(d.) $^2f(\underline{x})$ is positive semidefinite?
(e.) $^2f(\underline{x})$ is negative semidefinite?

(2.) While searching for the minimum of

$$f(x) = [x_1^2 + (x_2 + 1)^2]\,[\,x_1^2 + (x_2 - 1)^2\,]$$

$$= x_1^4 + x_2^4 + 2x_1^2x_2^2 + 2x_1^2 - 2x_2^2 + 1$$

with $\delta f/\delta x_1 = 4x_1^3 + 4\,x_1x_2^2 + 4x_1$
and $\delta f/\delta x_2 = 4x_2^3 + 4\,x_1^2x_2 - 4x_2$,

we terminate at the following points:

(a) x = (0,0)
(b) x = (0,1)
(c) x = (0, -1)
(d) x = (1,1)

Classify each point as

- local maximum
- local minimum
- neither of above

(3.) Consider again the unconstrained nonlinear programming problem of problem (2.). This problem was solved, using Newton's algorithm, the steepest descent algorithm, and Fletcher-Reeves algorithm, in each case starting at the point (1,1). (See attached pages for partial output.)

(a.) What is the initial search direction for Newton's method?
(b.) What is the initial search direction for the steepest descent method?
(c.) What is the initial search direction for the Fletcher-Reeves method?

(d.) What is the search direction at the second iteration of the Fletcher-Reeves method?

(4.) Consider the nonlinear program

$$\text{Minimize } f(x) = x_1^2 + 2x_2^2$$
$$\text{subject to } x_1^2 + x_2^2 \le 5$$
$$2x_1 - 2x_2 = 1$$

(a.) Write the Kuhn-Tucker conditions for this problem

(b.) Using the Kuhn-Tucker conditions, what can you say about the following points?

$(0,0)$ $\quad$ $(1, {}^1/_2)$ $\quad$ $({}^1/_3 , -{}^1/_6)$

(5.) Consider the nonlinear programming problem

$$\text{Minimize } f(x) = (x_1 - x_2)^2 + x_2$$
$$\text{subject to } g_1(x) = - x_1 - x_2 + 1 \le 0$$
$$g_2(x) = x_1 - 2x_2 - 3 \le 0$$
$$x_1 \ge 0, \; x_2 \ge 0$$

with initial point $\underline{x} = (3,0)$. This problem was solved by the feasible directions algorithm, gradient projection algorithm, and the generalized reduced gradient algorithm. (See attached pages for partial output.)

(a.) Write down the LP problem which is solved at the first iteration of the feasible directions algorithm.

(b.) Why, in the first iteration of the gradient projection method, when a search direction of (approximately) zero was computed, did the algorithm not terminate?

(c.) What is the search direction finally selected in the first iteration of the gradient projection method?

(d.) In the second iteration of the GRG algorithm, why was the Dependent Variable Set changed?

(e.) At the optimal solution, which constraints are tight? What are the values of the Lagrange multipliers which, with x^*, satisfy the Kuhn-Tucker conditions?